

# Core Collapse Supernovae: Trying to Explode Stars at NERSC

F. Douglas Swesty & Eric S. Myra

Terascale Supernova Initiative

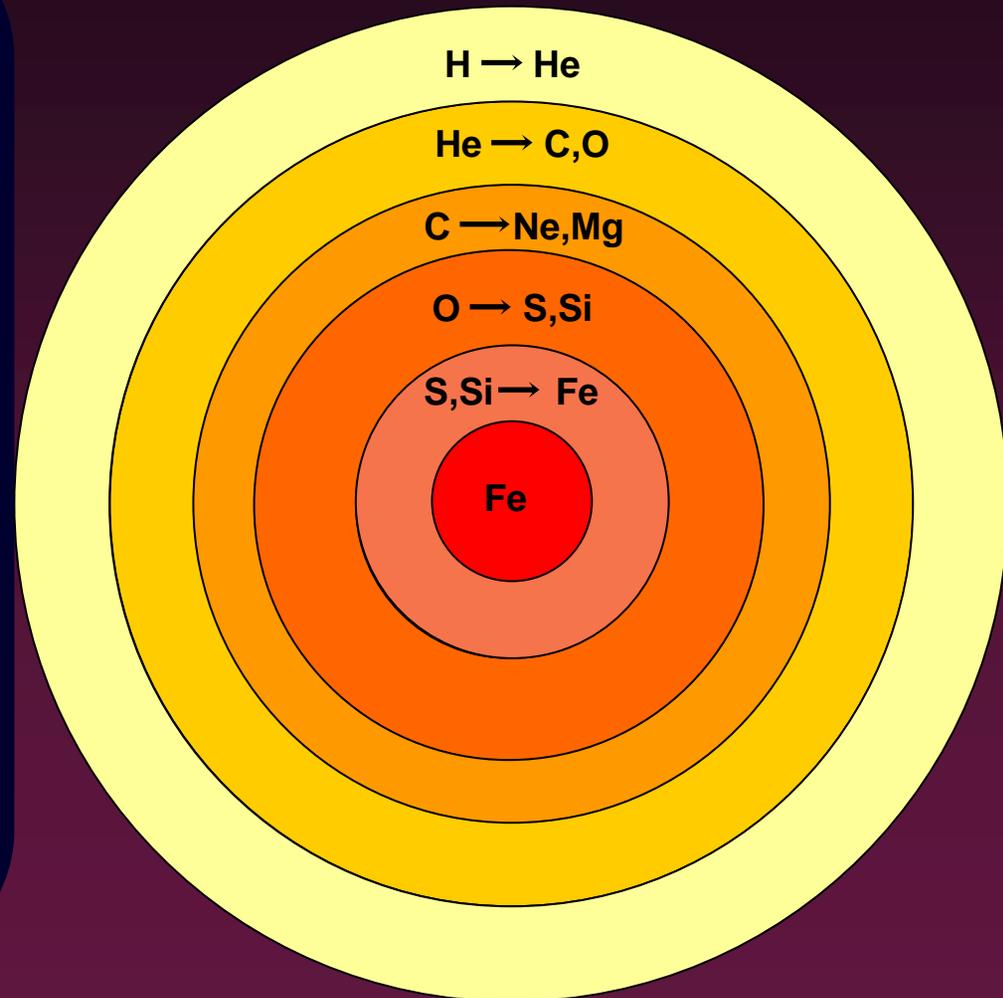
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Department of Physics & Astronomy  
STATE UNIVERSITY OF NY @ STONY BROOK

# Progenitor Evolution

## Stellar Core Collapse

- Massive stars evolve by burning lighter elements, by thermonuclear fusion, into heavier elements
- High mass stars will produce elements up to Silicon & Sulfur, which then burn into Iron
- Star has an Onionskin-like structure with layers of successively heavier elements
- Each burning stage progresses more rapidly
- When the iron core mass becomes about 1.2-1.4 solar masses the core can no longer sustain itself against the pull of gravity & it collapses



# "Prompt" Epoch

## Core Collapse & Bounce

-Iron core collapses in 1/100<sup>th</sup> of a second

-Neutrinos are produced:  $e^- + p \rightarrow n + \bar{\nu}_e$

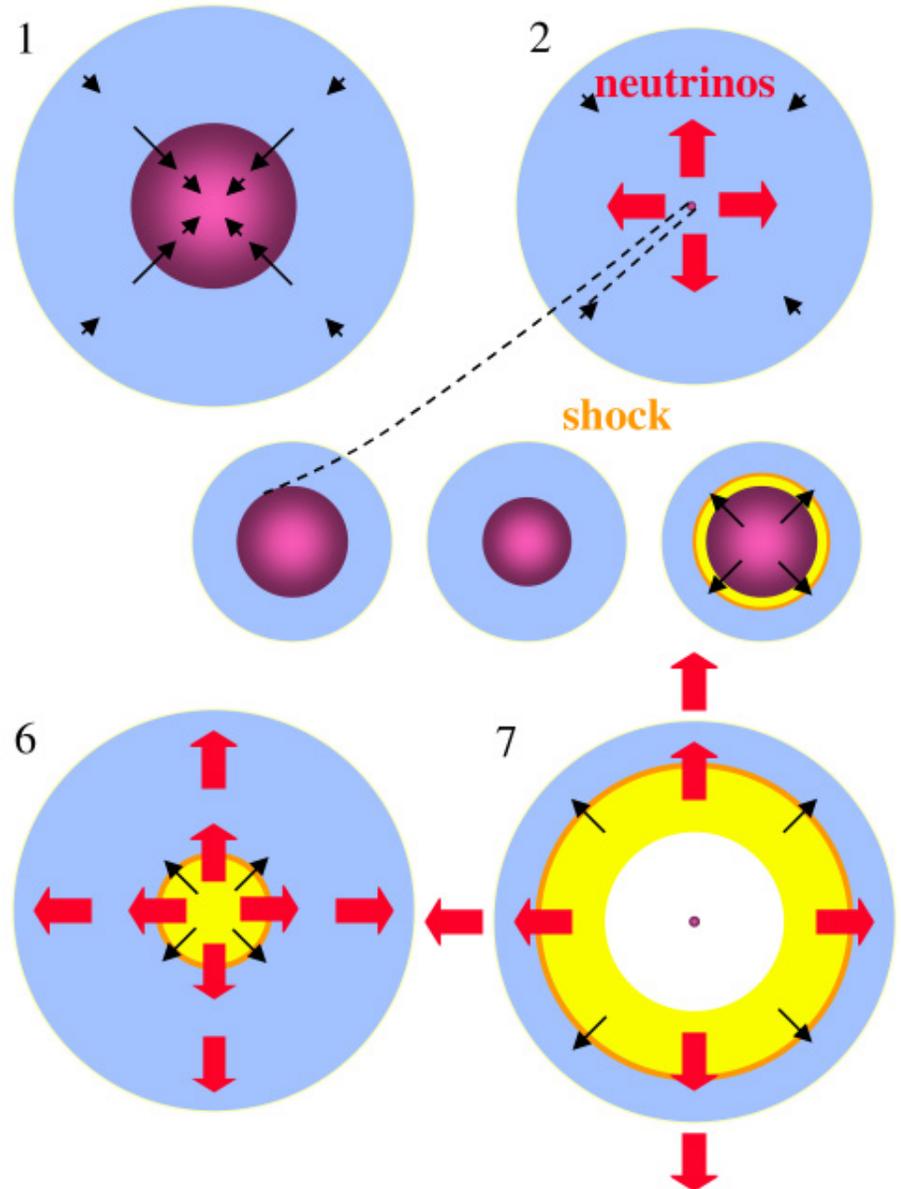
-Neutrinos escape from the collapsing core until the material becomes too dense

-The core collapses until densities of about  $2.5 \times 10^{14} \text{ g/cm}^3$  is reached

-Stiff nuclear forces cause the collapse to halt and the core rebounds outward

-Outward moving core forms a shock wave when it hits the exterior regions where matter is still falling inward

## Core Collapse and Explosion



# Convective Shock Revitalization Epoch

## Supernova Explosion

-Shock wave heats matter as it moves outward

-Hot matter produces neutrino/anti-neutrino pairs of three types



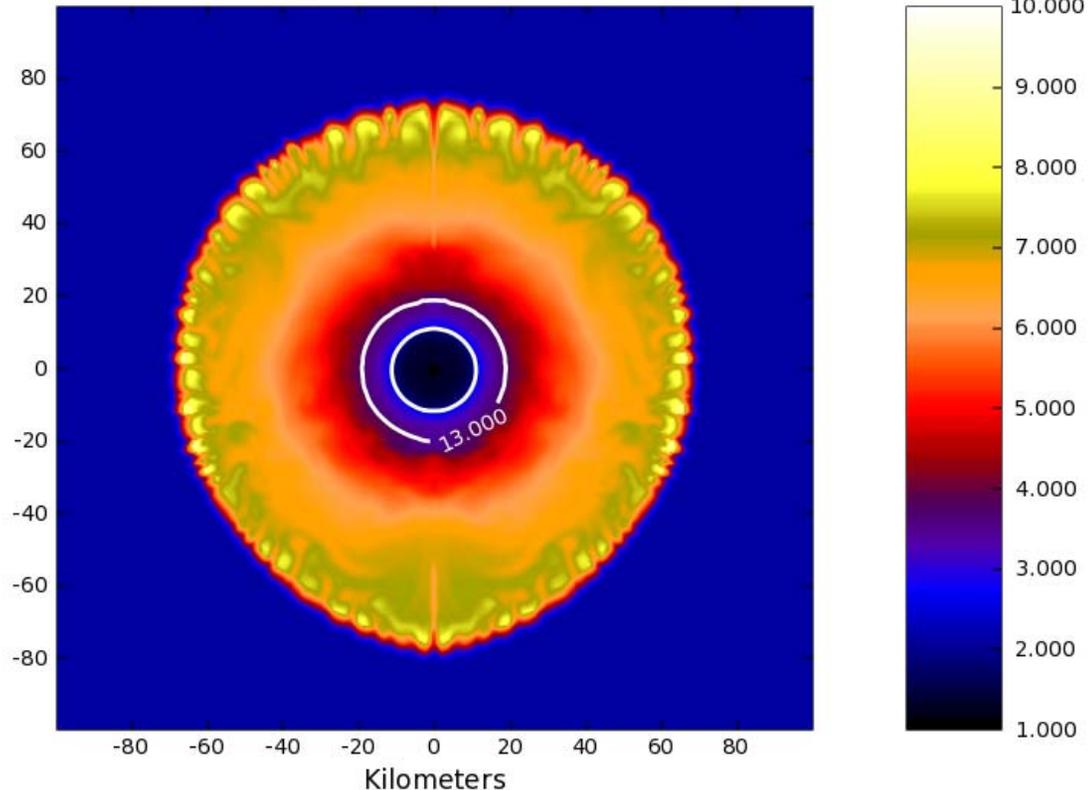
-Shock wave stalls before it reaches outer edge of iron core

-Hot (high entropy) matter starts rising upward (convection)

-A combination of convection and neutrino heating of the matter somehow revitalizes the stalled shock wave and causes the observed explosion

-Explosion leaves either a neutron star or a black hole behind

entropy & log10\_density Time = 253.837 msec



# Neutrino Radiation-Hydrodynamic Model

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad \frac{\partial (\rho Y_e)}{\partial t} + \nabla \cdot (\rho Y_e \mathbf{v}) = -m_b \sum_f \int d\epsilon \left( \frac{S_\epsilon}{\epsilon} - \frac{\bar{S}_\epsilon}{\epsilon} \right)$$

$$\frac{\partial E}{\partial t} + \nabla \cdot (E \mathbf{v}) + P \nabla \cdot \mathbf{v} = - \sum_f \int d\epsilon (S_\epsilon + \bar{S}_\epsilon) \quad E = f(T, \rho, Y_e)$$

$$P = g(T, \rho, Y_e)$$

$$\frac{\partial (\rho v_i)}{\partial t} + \nabla \cdot (\rho v_i \mathbf{v}) + (\nabla P)_i + \rho (\nabla \Phi)_i + \nabla \cdot \left\{ \sum_f \int d\epsilon (\chi_\epsilon E_\epsilon + \bar{\chi}_\epsilon \bar{E}_\epsilon) \right\} = 0 \quad \nabla^2 \Phi = 4\pi \rho G_N$$

$$\frac{\partial E_\epsilon}{\partial t} + \nabla \cdot (E_\epsilon \mathbf{v}) - \nabla \cdot (D_\epsilon \nabla E_\epsilon) - \epsilon \frac{\partial}{\partial \epsilon} (\chi_\epsilon E_\epsilon) : \nabla \mathbf{v} = S_\epsilon \quad 0 \leq E_\epsilon \leq \frac{\epsilon^3}{\alpha}$$

$$\frac{\partial \bar{E}_\epsilon}{\partial t} + \nabla \cdot (\bar{E}_\epsilon \mathbf{v}) - \nabla \cdot (\bar{D}_\epsilon \nabla \bar{E}_\epsilon) - \epsilon \frac{\partial}{\partial \epsilon} (\bar{\chi}_\epsilon \bar{E}_\epsilon) : \nabla \mathbf{v} = \bar{S}_\epsilon \quad 0 \leq \bar{E}_\epsilon \leq \frac{\epsilon^3}{\alpha}$$

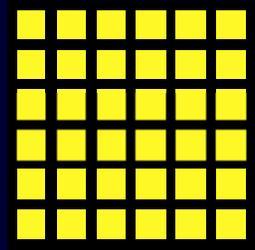
$$S_\epsilon = S_\epsilon \left( 1 - \frac{\alpha}{\epsilon^3} E_\epsilon \right) - c \kappa_\epsilon^a E_\epsilon + \left( 1 - \frac{\alpha}{\epsilon^3} E_\epsilon \right) c \int d\epsilon' \kappa^s(\epsilon', \epsilon) E_{\epsilon'} \\ - E_\epsilon c \int d\epsilon' \kappa^s(\epsilon, \epsilon') \left( 1 - \frac{\alpha}{\epsilon'^3} E_{\epsilon'} \right) + \left( 1 - \frac{\alpha}{\epsilon^3} E_\epsilon \right) \epsilon \int d\epsilon' G(\epsilon, \epsilon') \left( 1 - \frac{\alpha}{\epsilon'^3} \bar{E}_{\epsilon'} \right)$$

$$\bar{S}_\epsilon = \bar{S}_\epsilon \left( 1 - \frac{\alpha}{\epsilon^3} \bar{E}_\epsilon \right) - c \bar{\kappa}_\epsilon^a \bar{E}_\epsilon + \left( 1 - \frac{\alpha}{\epsilon^3} \bar{E}_\epsilon \right) c \int d\epsilon' \bar{\kappa}^s(\epsilon', \epsilon) \bar{E}_{\epsilon'} \\ - \bar{E}_\epsilon c \int d\epsilon' \bar{\kappa}^s(\epsilon, \epsilon') \left( 1 - \frac{\alpha}{\epsilon'^3} \bar{E}_{\epsilon'} \right) + \left( 1 - \frac{\alpha}{\epsilon^3} \bar{E}_\epsilon \right) \epsilon \int d\epsilon' G(\epsilon', \epsilon) \left( 1 - \frac{\alpha}{\epsilon'^3} E_{\epsilon'} \right)$$

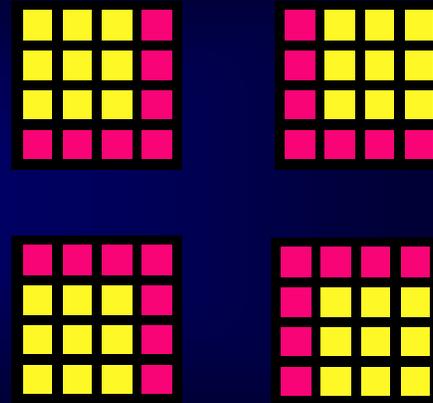
# Parallel Implementation

Global Problem Domain

Problem split up into  
one spatial sub-domain  
per processor



Spatial  
domain  
decomposition



Adjacent  
processes in  
Cartesian  
topology must  
exchange  
ghost zones  
when needed  
by algorithms

- ❑ Code (V2D) written in F95
- ❑ Continuous process of Verification & Validation
- ❑ Componentized architecture
- ❑ Message passing done in MPI
- ❑ Parallel I/O done with parallel HDF5
- ❑ Scalable to at least 1024-2048 processors for "medium sized" problems

# Implicit Neutrino Transport w/ Newton-Krylov Iteration

N-K solution accomplished via Newton-BiCGSTAB

Currently using sparse parallel approximate inverse preconditioning (physics based) -- highly parallel

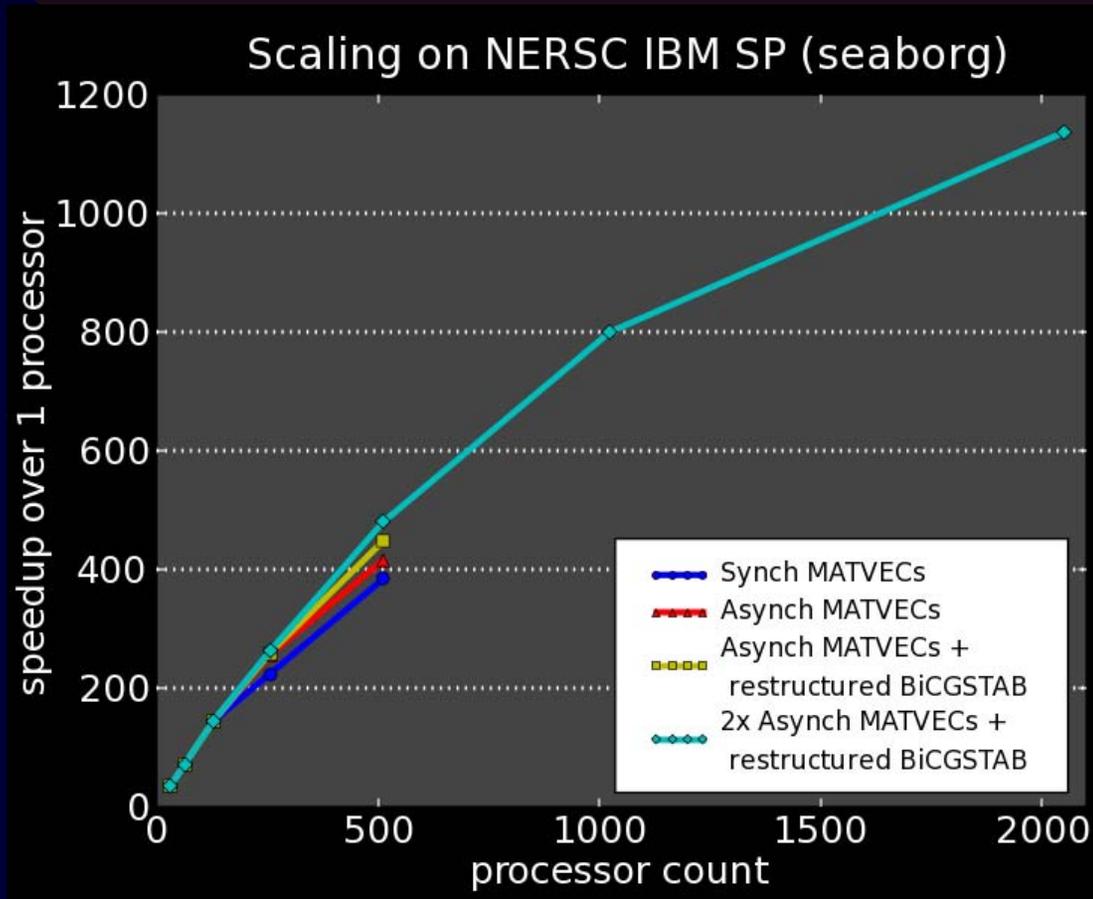
MATVECS can be carried out asynchronously to hide communication

BiCGSTAB can be restructured to reduce global reductions

Typically 10-20 Bicgstab iterations per Newton iteration; 2-3 Newton iterations

Implicit solvers have to be ultra-fast -- few  $\times 10^5$  timesteps to complete simulation!

Currently Incorporating TOPS SUNDIALS solvers into V2D for fully implicit radiation-hydro



# Data Management & Networking Challenges

- ❑ Couldn't handle the data without LBONE Depots & LORS Tools!
- ❑ Scientific Process Automation (SPA) systems is easing workflow management tremendously!



75 msec



## Models of the Convective Epoch

- Prior to a few years ago state of the art was:
  - 2-D models with gray transport approximation (produced explosions; assumed distribution function; approximated rates & opacities; no dynamic diffusion in Eulerian models)
  - Uncoupled radial-ray multi-group models (saw no explosions; lacked full coupling; no lateral movement of neutrinos; no dynamic diffusion)
- More Recently:
  - Fully coupled 2-D radiation-hydrodynamic MGFLD models
  - Partially coupled 2-D Multigroup Variable Eddington Factor models
  - Fully coupled 2-D Boltzmann transport models (coming soon!)

### Baseline Model

-S15S7B progenitor model of Woosley & Weaver

-Neutrino Physics at the level of Bruenn (1985) w/ NES turned off

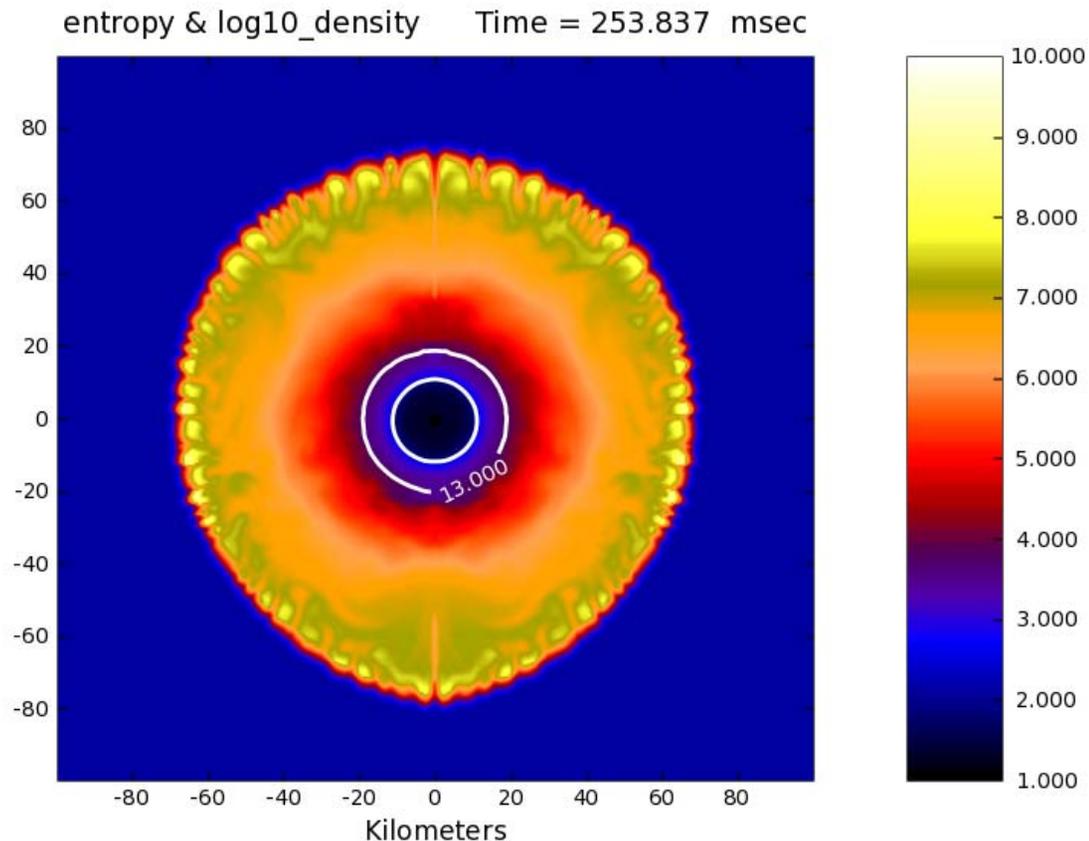
-EOS is LS K=180 MeV model

-Collapse run to  $\rho=10^{14}$  g/cm<sup>3</sup> w/ 1-D Lagrangean code

-Baseline model does not explode on timescales of 35 msec

## Early Onset of Convection

- Convection begins to occur early when 2-D models are initiated from bounce
  - Richtmyer-Meshkov (impulsive limit of Raleigh-Taylor) instability
  - Before shock gets out to  $> 100$  Km
- At 256x256 spatial resolution we see much smaller scale convective flow features than other groups have reported
  - Fully resolved? Probably not...



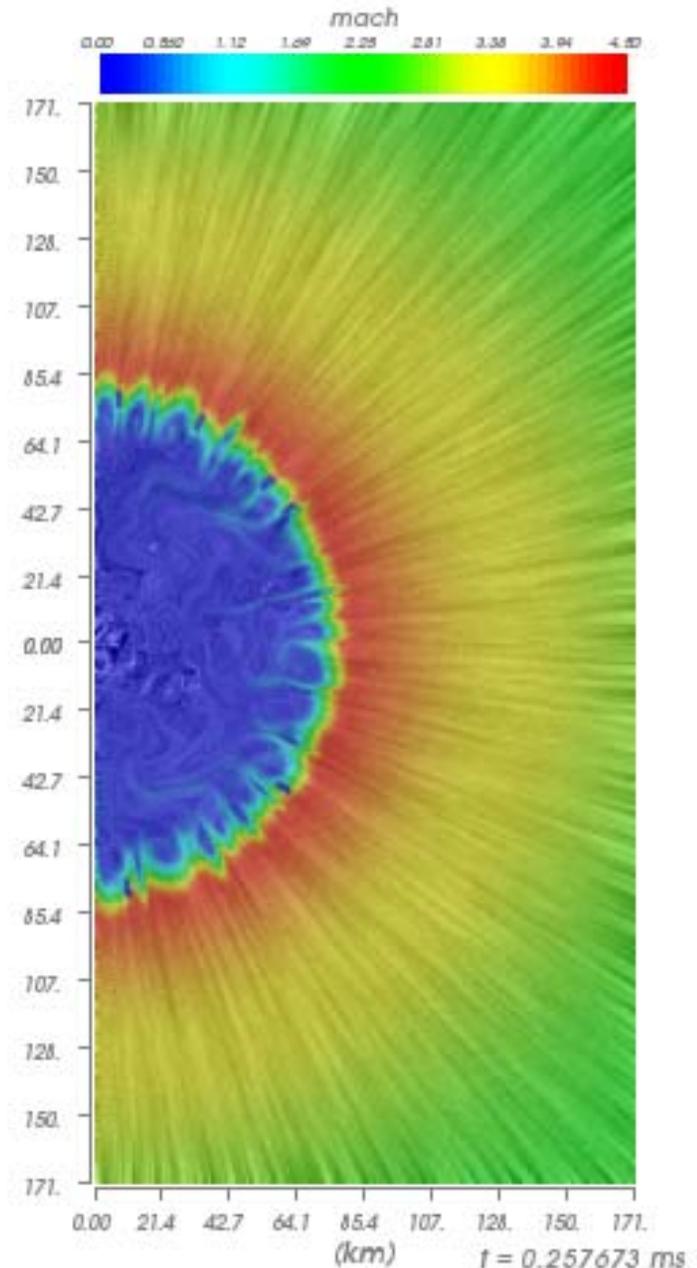
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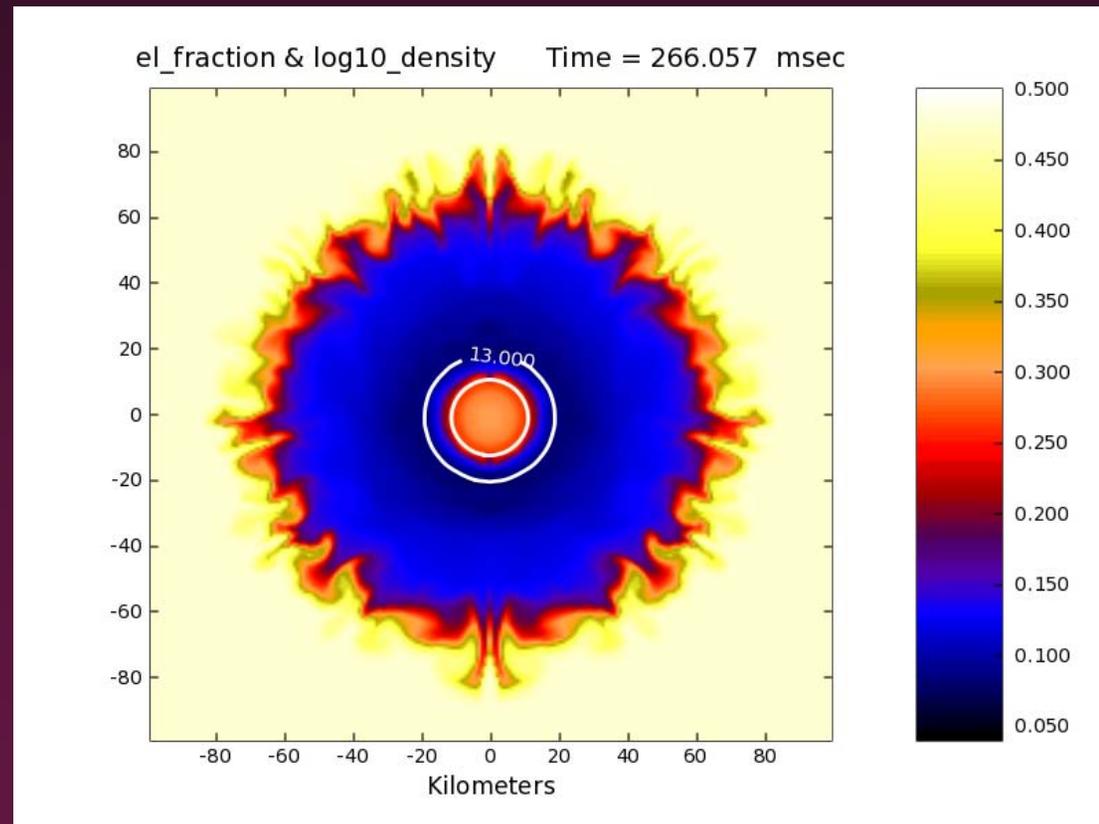
## Behavior of Baseline Model

- ❑ Shock stagnates on timescales of 30 msec
- ❑ Shock is strong; flow into it @ Mach 4
- ❑ Shock initially shows large scale SASI mode which dies out & shock circularizes
- ❑ Could this reinitialize at later times?
- ❑ We see convection in both:
  - ❑ The proto-neutron star (in optically thick regions below the neutrinosphere)
  - ❑ Between stalled shock & neutrinosphere
- ❑ Vigorous Proto-Neutron Star convection is entropy driven and is short-lived
- ❑ Convection above the neutrinosphere maintains and is fueled by neutrino heating of base of the layer



## Behavior of $Y_e$ in Optically Translucent Regions

- Convections maintains  $Y_e$  (electron fraction) at a much higher level in regions around neutrinosphere
  - Convection cycles in high  $Y_e$  (electron fraction) fuel into rapid deleptonization region
  - 1-D calcs show  $Y_e$  down at 0.05 within 5-10 msec.
- Very few heavy nuclei below the shock
- Little effect on opacity



# Proto-neutron star (PNS) convection

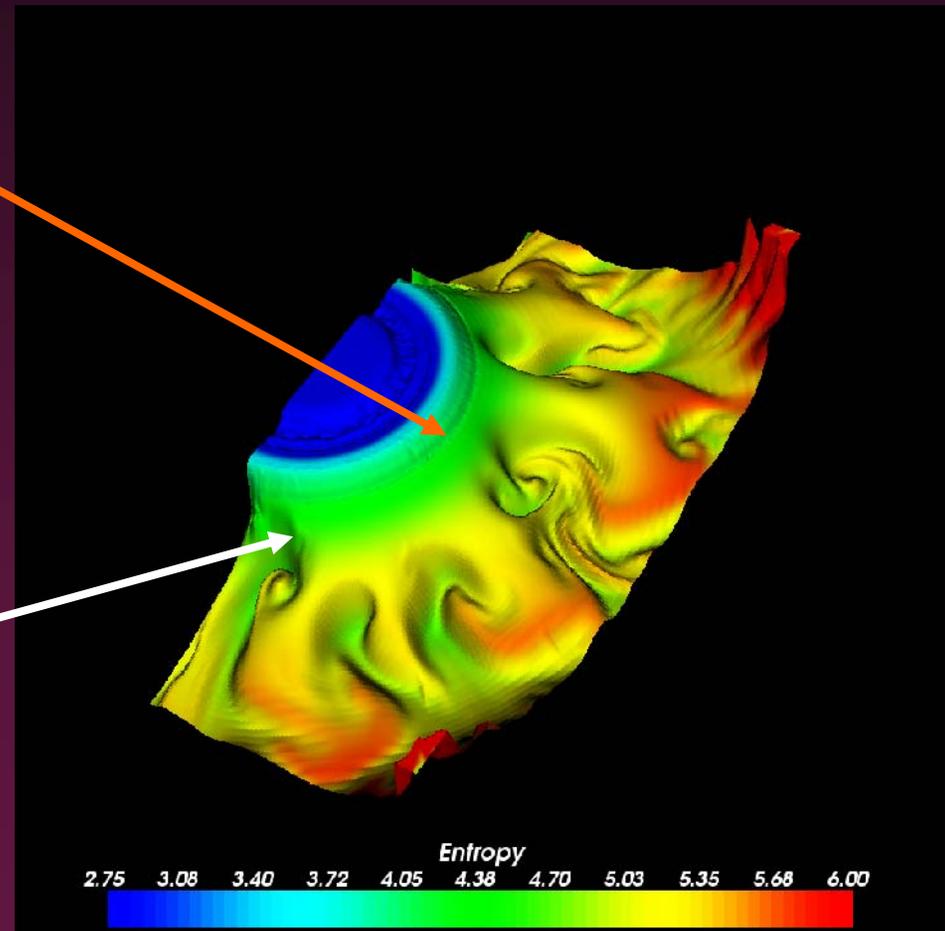
Vigorous convection takes place in regions where both the electron fraction and entropy have steep gradients

The convection we see at the surface of the neutron star seems to be entropy driven.

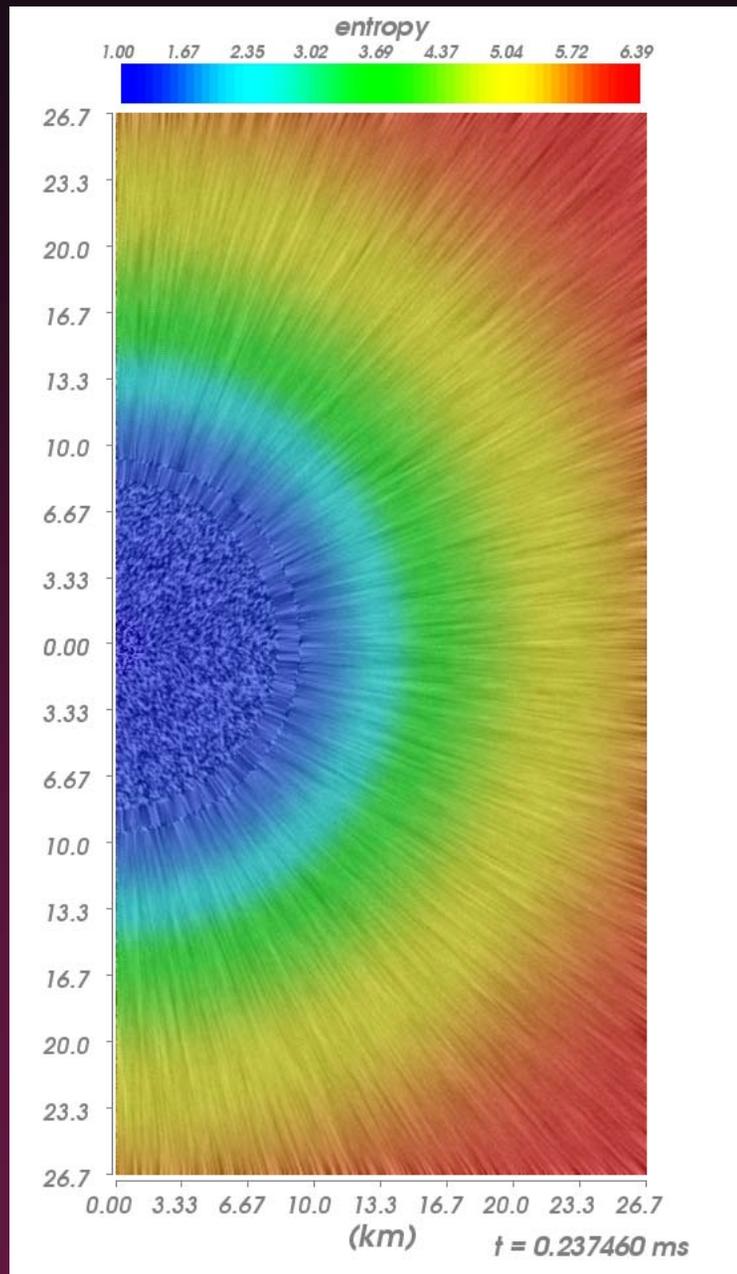
Further analysis is underway

The vigorous convective zone seems to end at the neutron star surface. But...

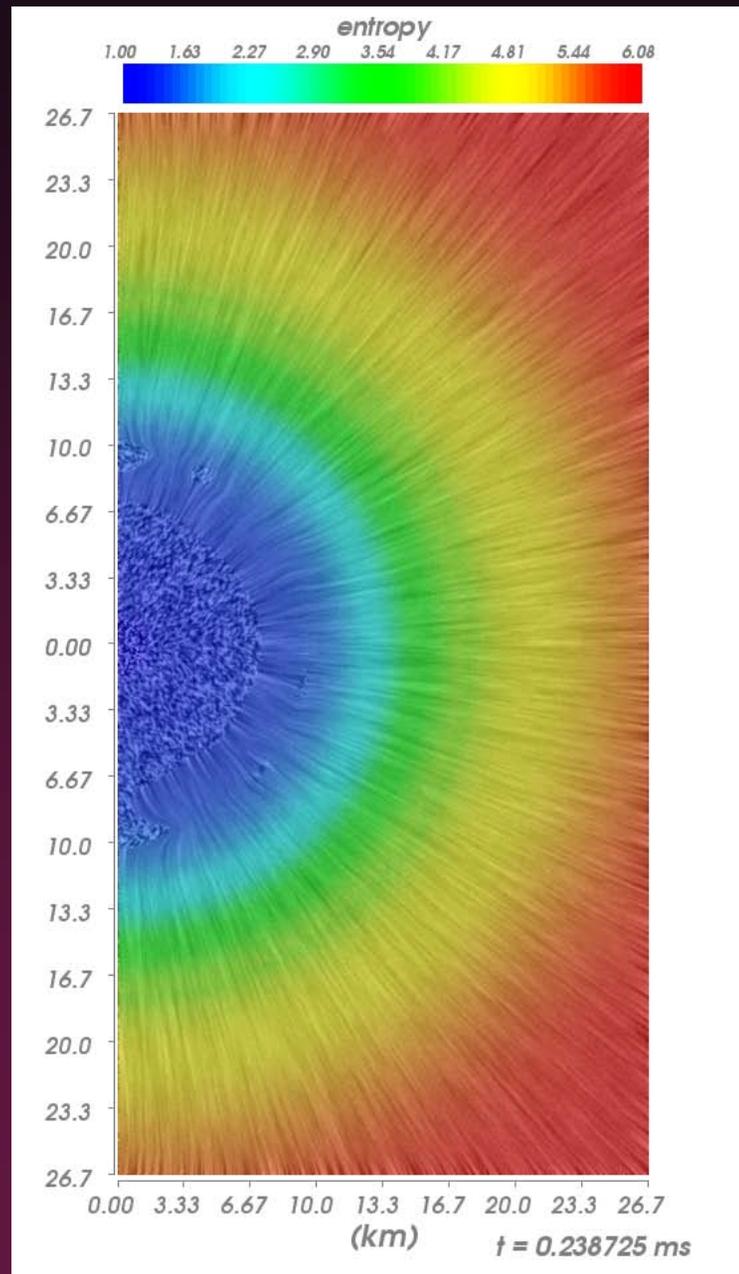
Colormap depicts entropy while height illustrates electron fraction



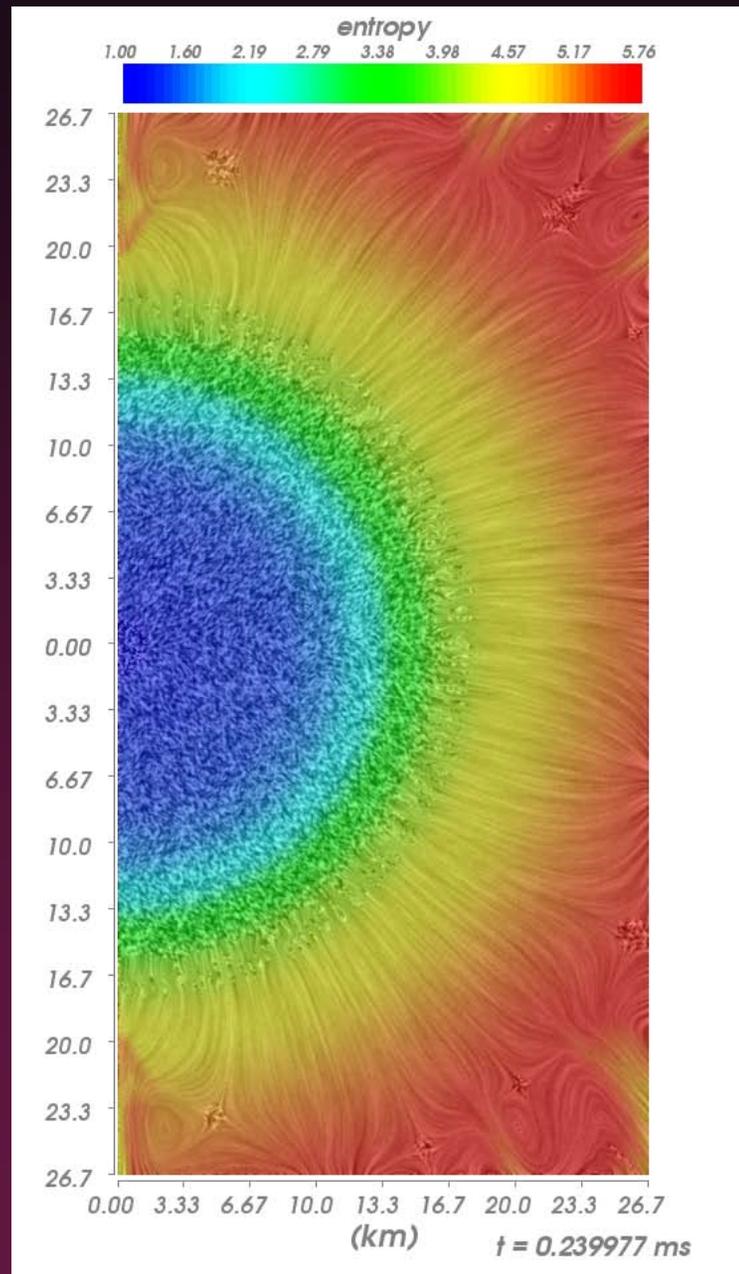
# *Proto-neutron star (PNS) convection*



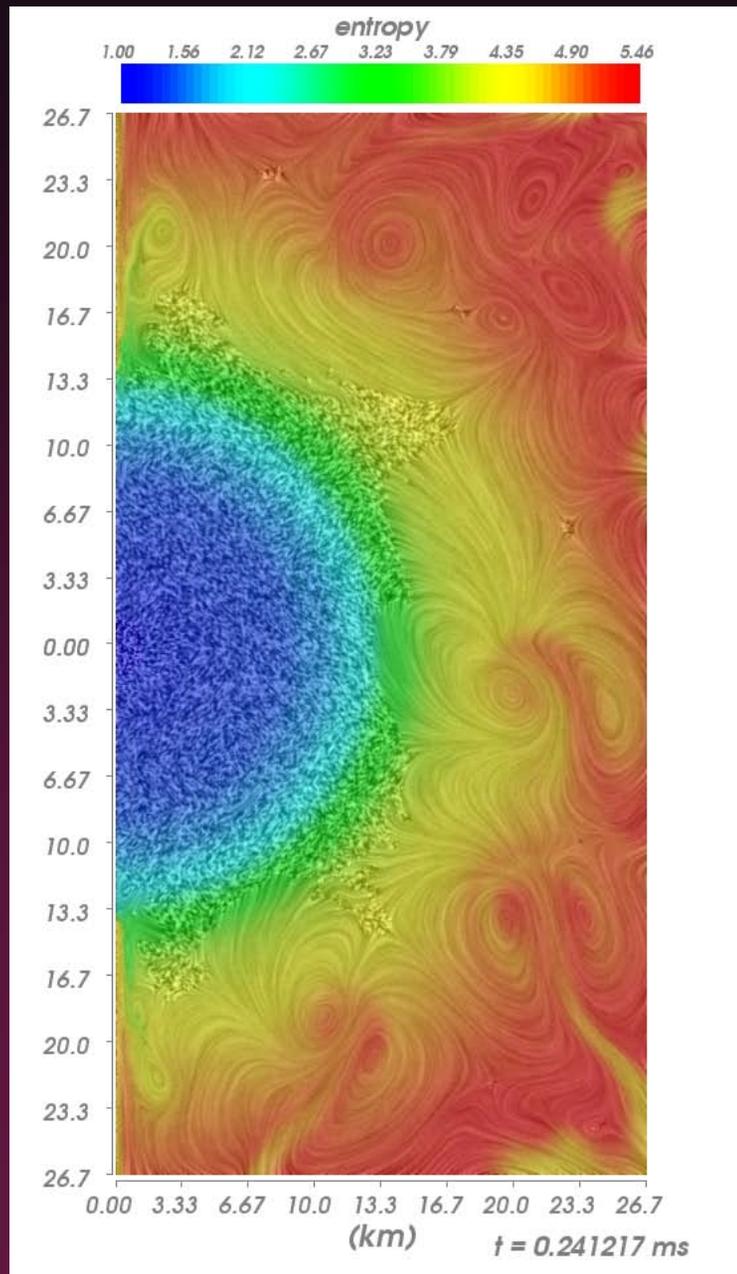
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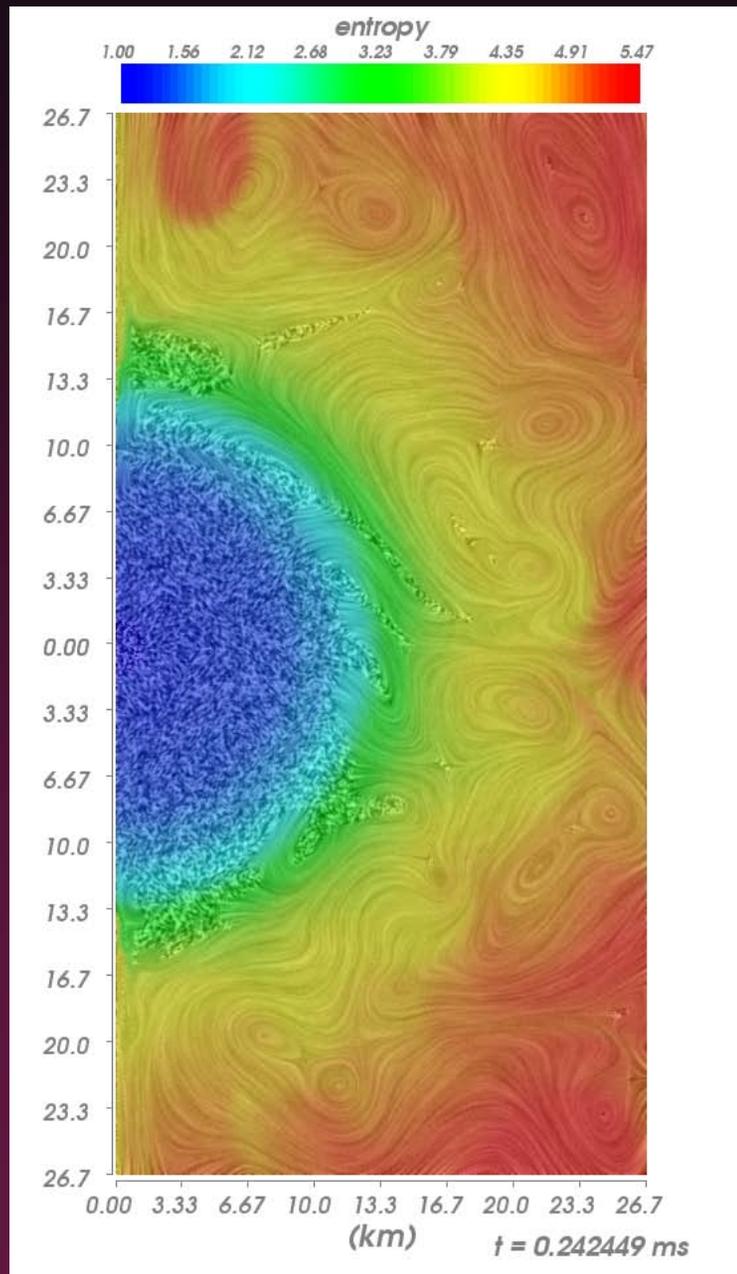
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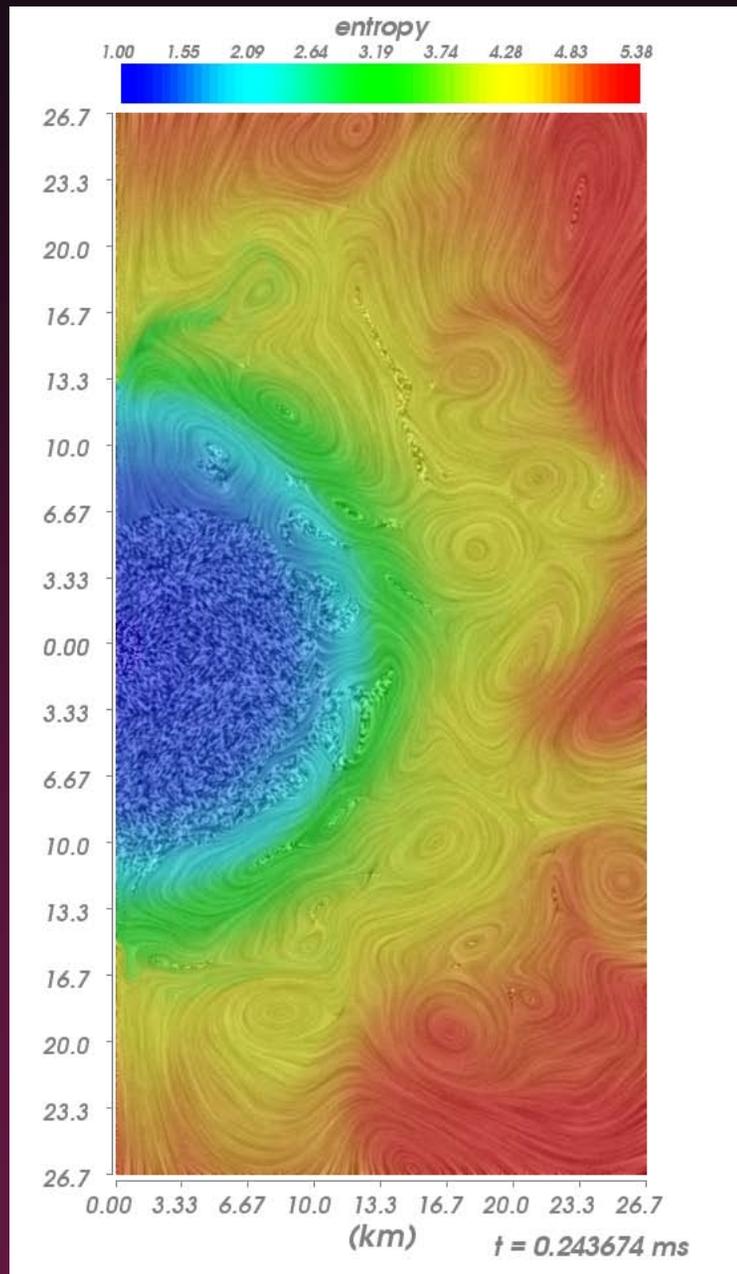
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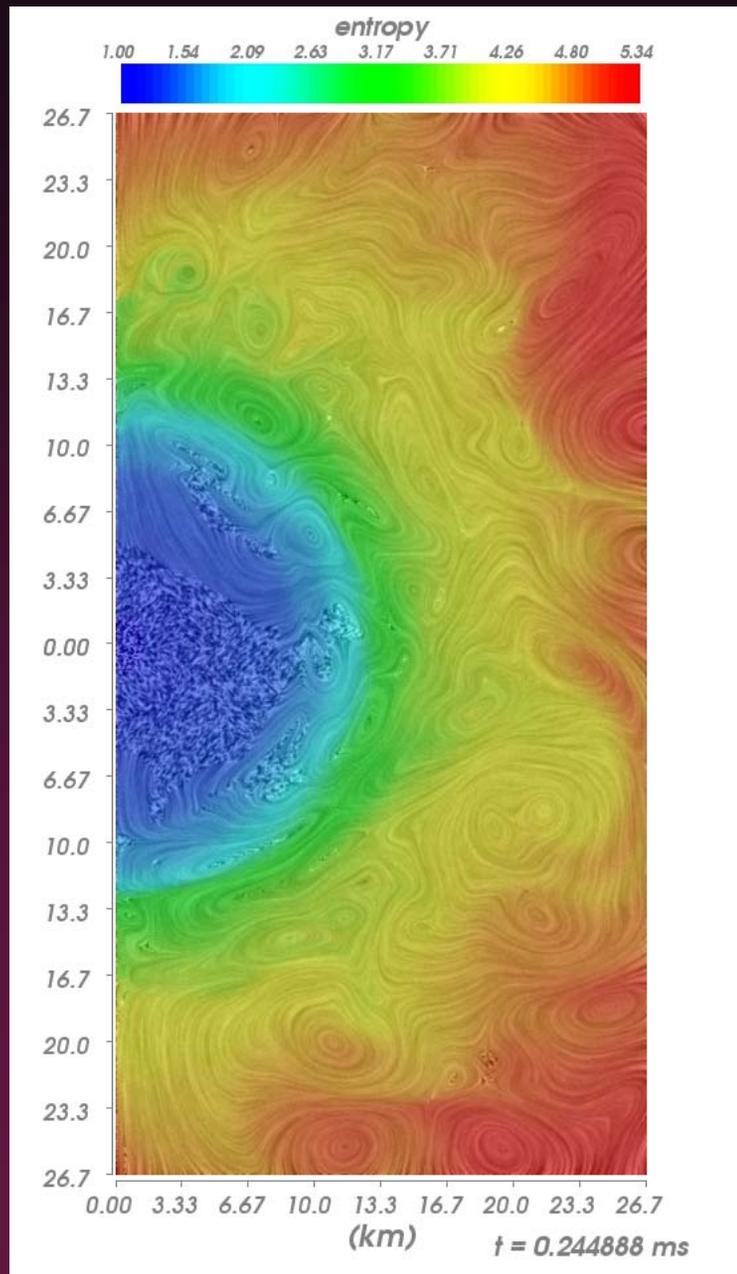
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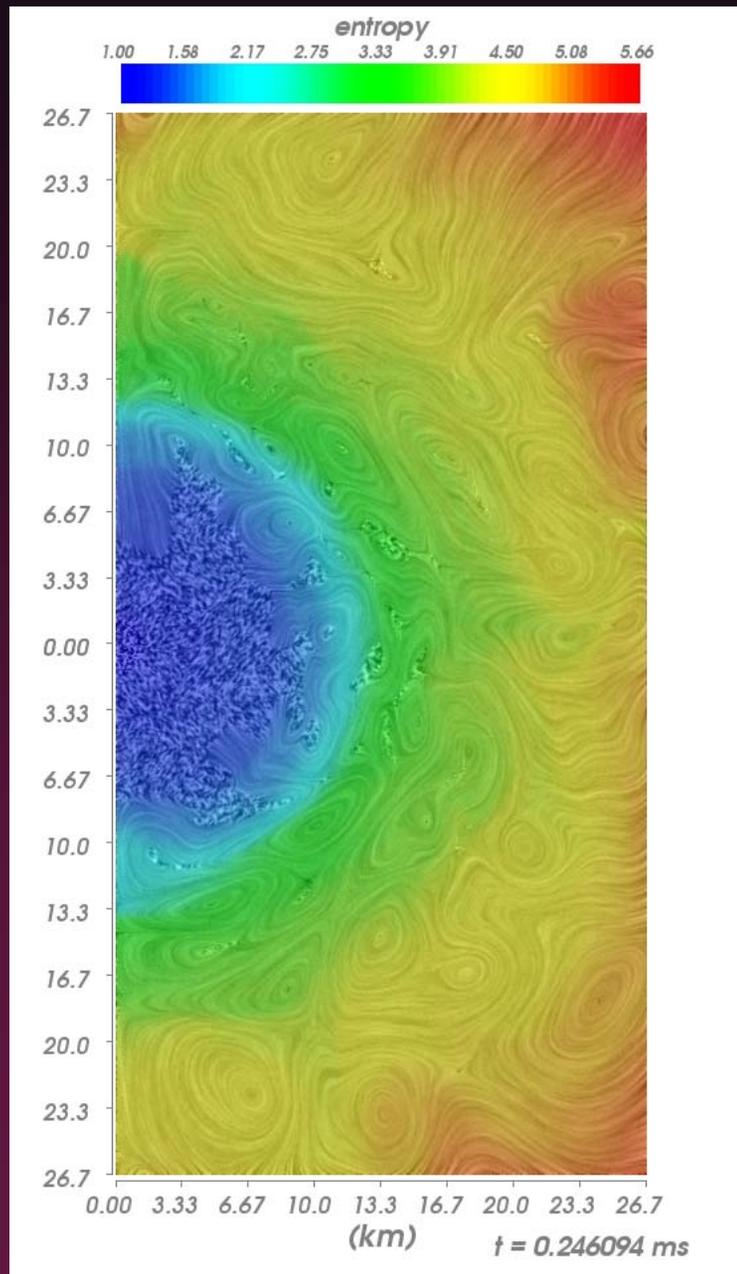
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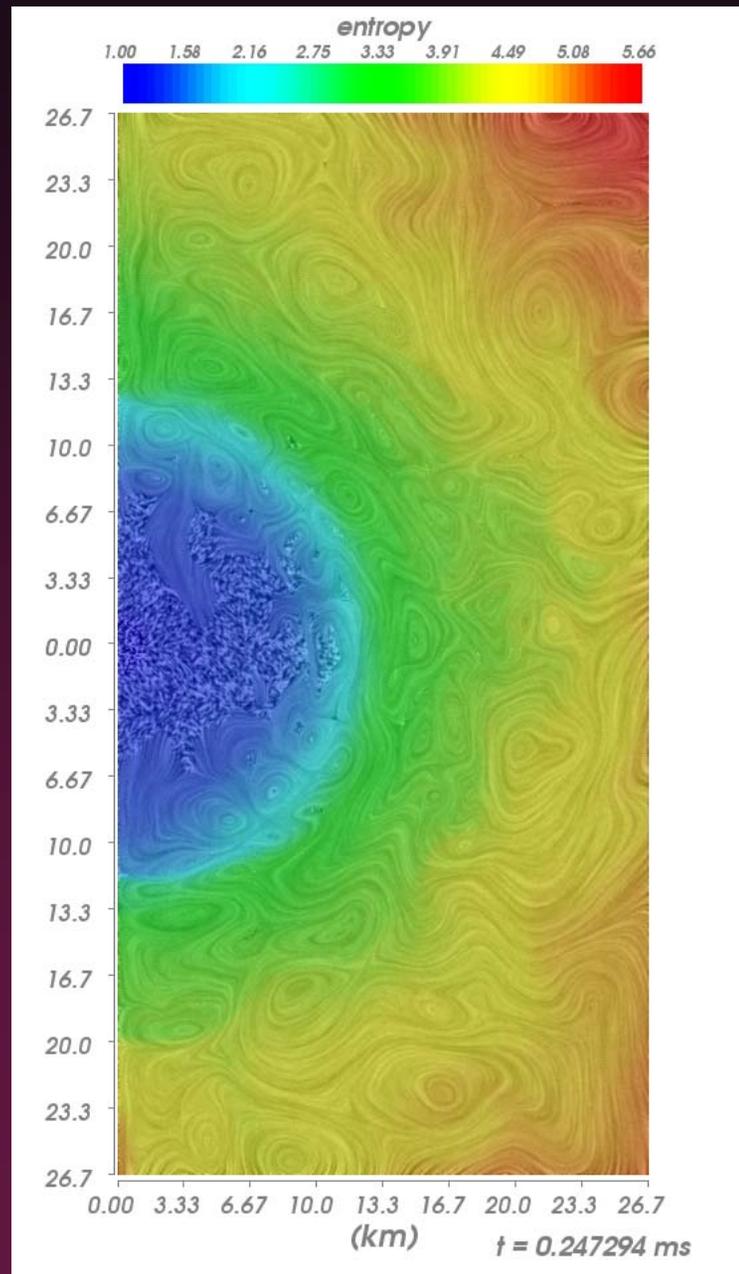
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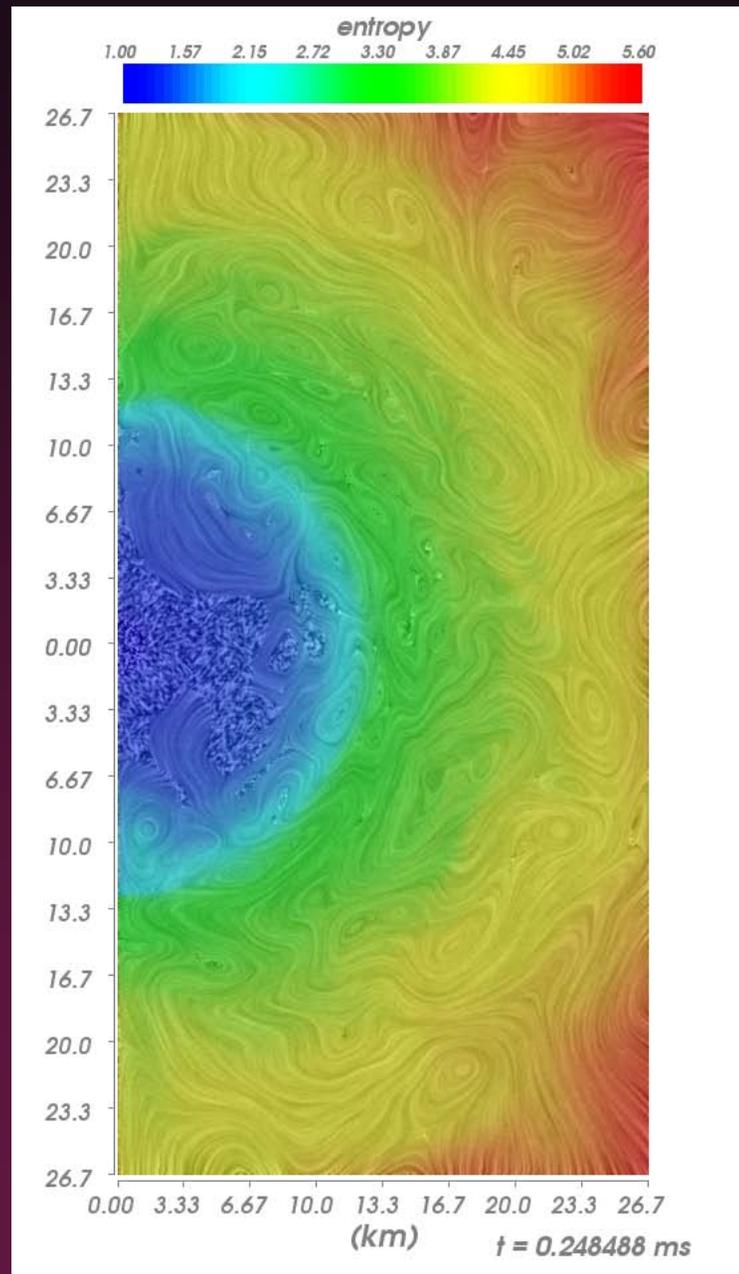
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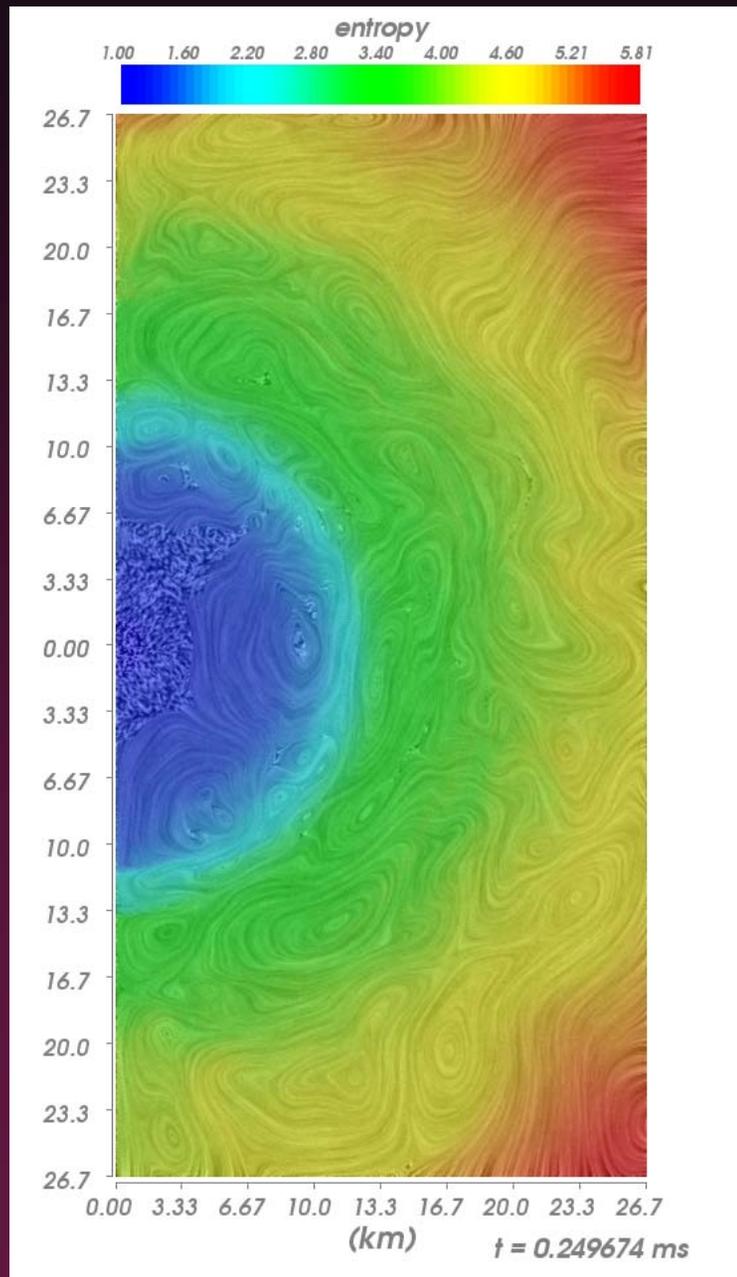
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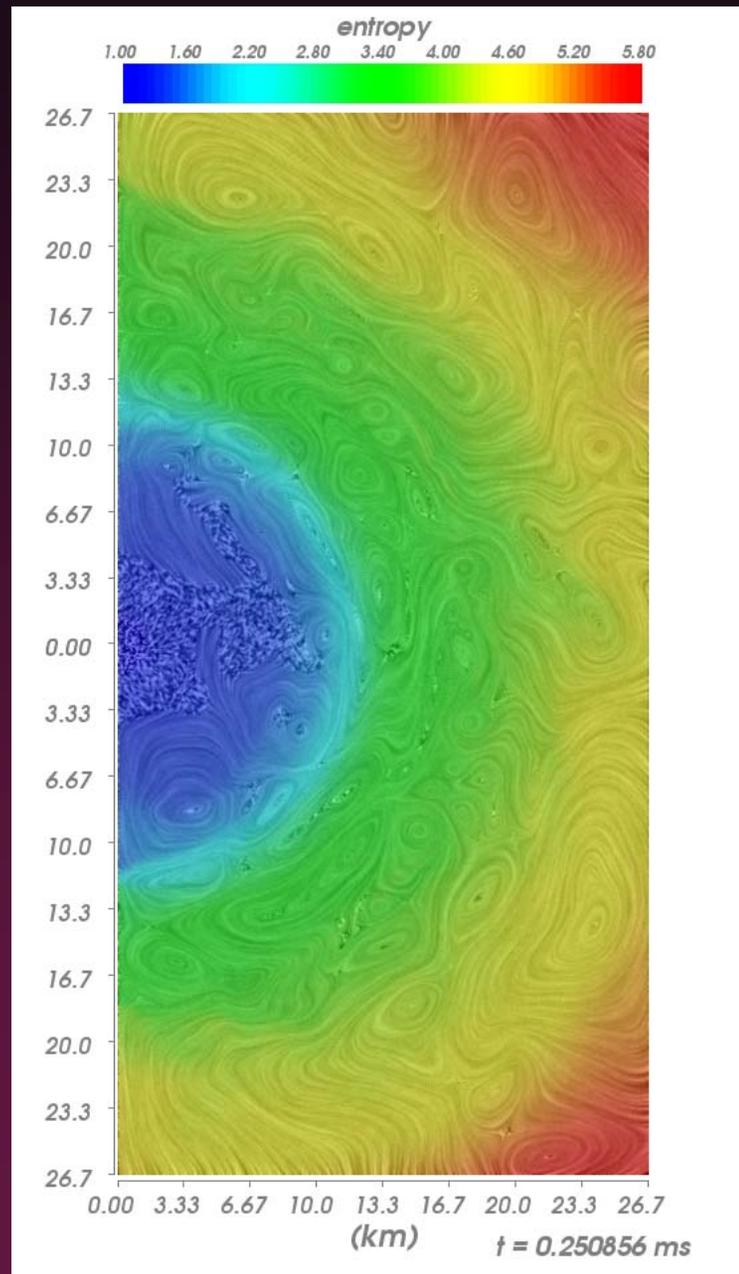
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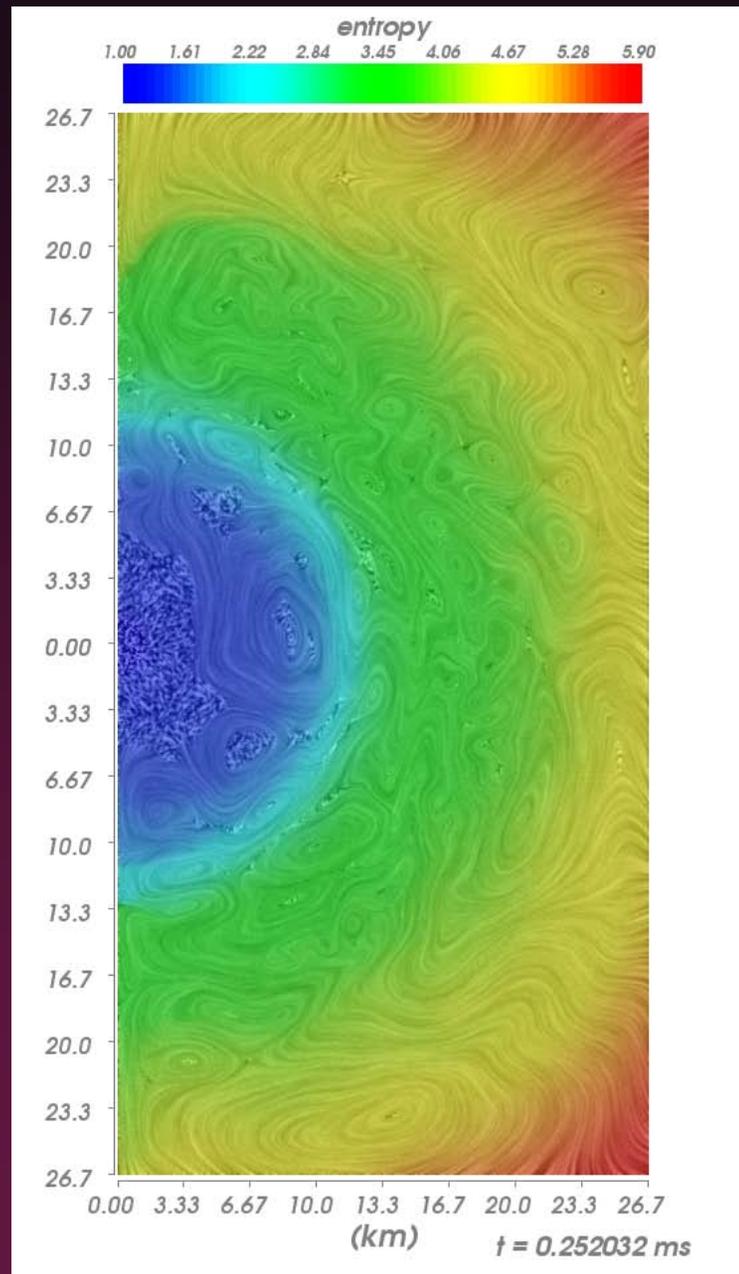
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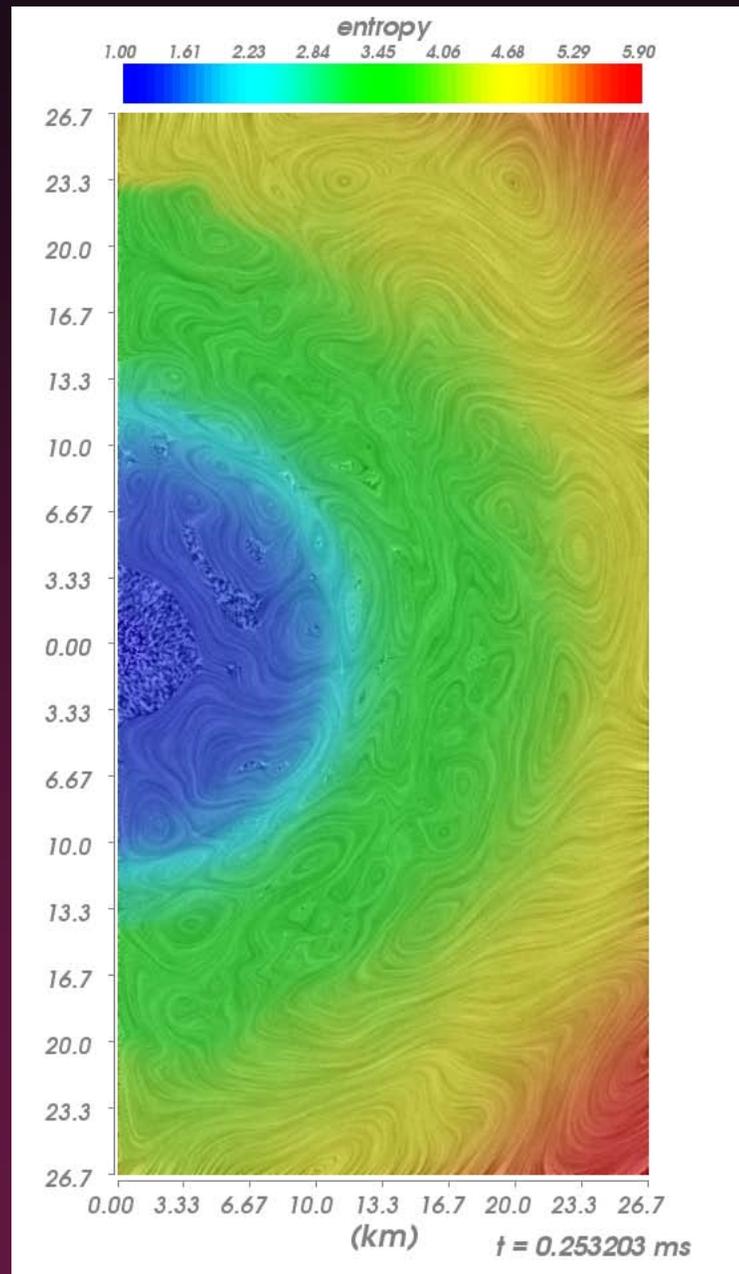
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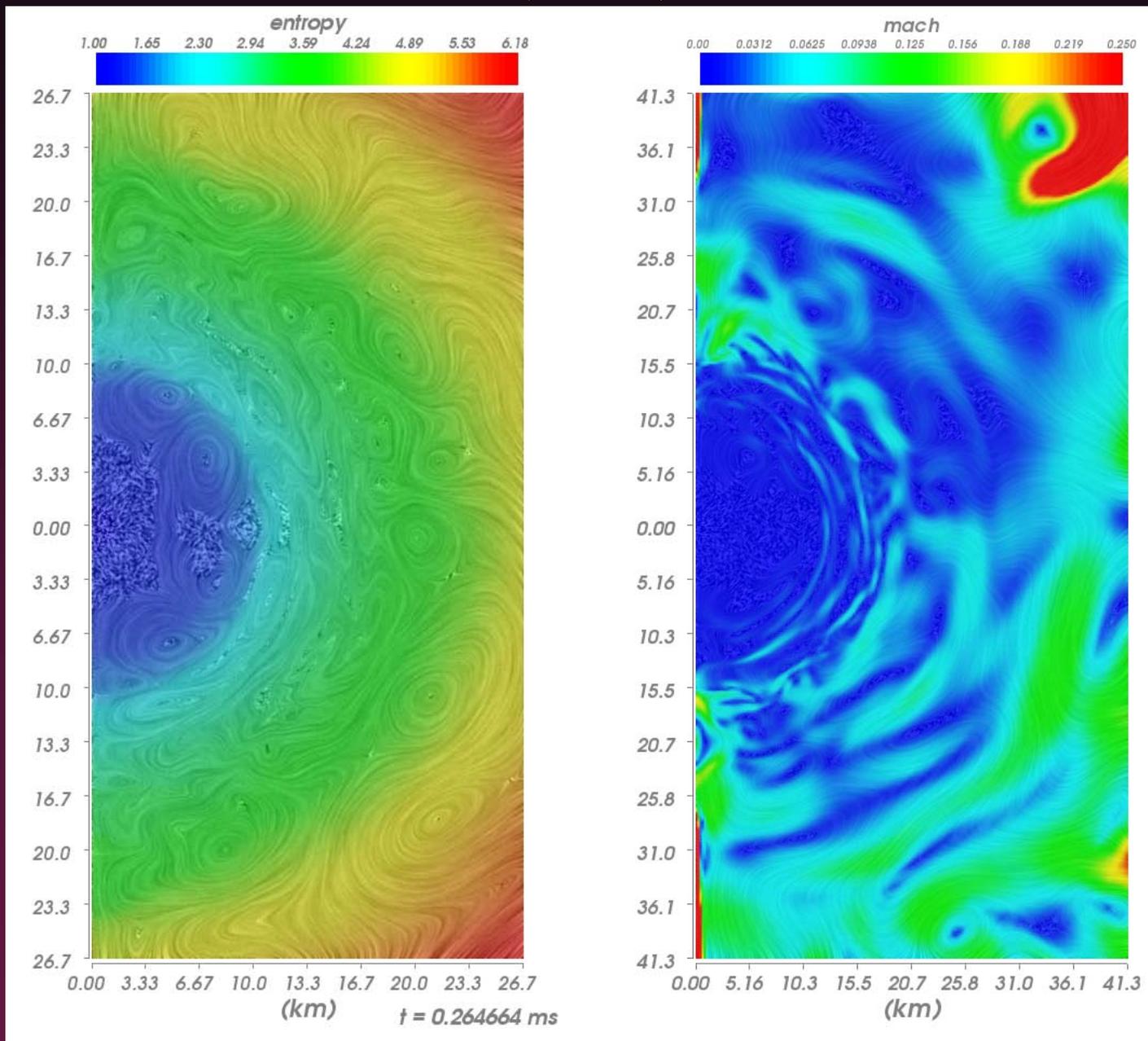
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# Proto-neutron star (PNS) convection



# $O(v/c)$ Multigroup Neutrino Energy Transport

One pair of neutrino/anti-neutrino energy eqns. for each of three neutrino

flavors:  $e, \mu,$

$$\left\{ \begin{aligned} \frac{\partial E_\epsilon}{\partial t} + \nabla \cdot (E_\epsilon \mathbf{v}) + \nabla \cdot \mathbf{F}_\epsilon - \epsilon \frac{\partial}{\partial \epsilon} (P_\epsilon) : \nabla \mathbf{v} &= \mathcal{S}_\epsilon \\ \frac{\partial \bar{E}_\epsilon}{\partial t} + \nabla \cdot (\bar{E}_\epsilon \mathbf{v}) + \nabla \cdot \bar{\mathbf{F}}_\epsilon - \epsilon \frac{\partial}{\partial \epsilon} (\bar{P}_\epsilon) : \nabla \mathbf{v} &= \bar{\mathcal{S}}_\epsilon \end{aligned} \right.$$

Energy density =  $E_\epsilon \equiv \frac{1}{c} \int I_\epsilon d\Omega$       Pressure tensor =  $P_\epsilon \equiv \frac{1}{c} \int I_\epsilon \Omega^2 d\Omega$

Flux =  $\mathbf{F}_\epsilon \equiv \int I_\epsilon \Omega d\Omega$

## Flux-Limited Diffusion (FLD) Approximation

Diffusion coefficient

$$\begin{aligned} \mathbf{F}_\epsilon &= -D_\epsilon \nabla E_\epsilon & \bar{\mathbf{F}}_\epsilon &= -\bar{D}_\epsilon \nabla \bar{E}_\epsilon \\ P_\epsilon &= \chi_\epsilon E_\epsilon & \bar{P}_\epsilon &= \bar{\chi}_\epsilon \bar{E}_\epsilon \end{aligned}$$

Optically thin limit      Optically thick limit

$$D_\epsilon \rightarrow \frac{-c E_\epsilon}{|\nabla E_\epsilon|}$$

$$D_\epsilon \rightarrow \frac{c}{3\kappa_\epsilon}$$

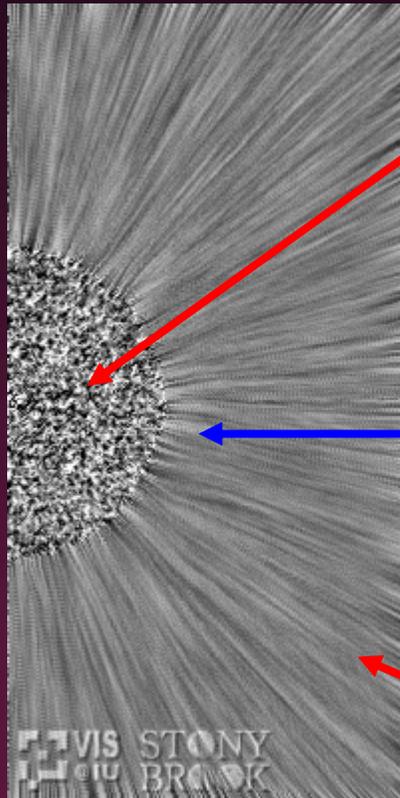
$$\chi_\epsilon \rightarrow 1$$

$$\chi_\epsilon \rightarrow \frac{1}{3}$$

Tensor Eddington factor



*Dynamic diffusion (via advective flux) plays an important role in getting neutrinos out of the proto-neutron star core. Dynamic diffusion has thus far been neglected in other 2-D & 3-D supernova simulations.*



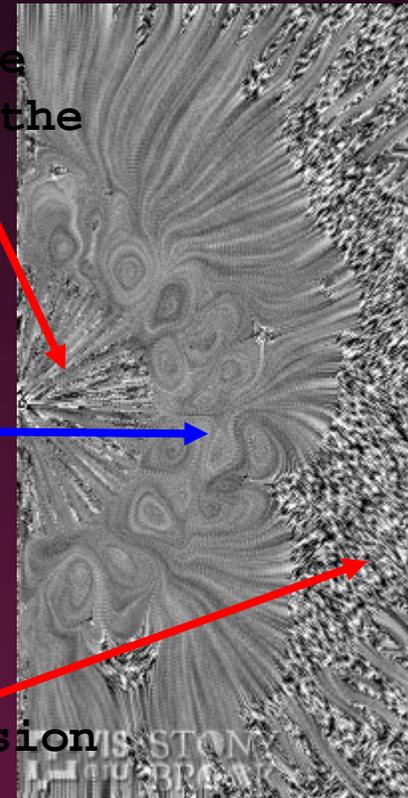
Radiative flux

$$\mathbf{F}_\epsilon = -D_\epsilon \nabla E_\epsilon$$

Diffusive region where dynamic diffusion is the dominant effect

Optically translucent region where radiative & dynamic diffusion compete

Free-streaming region where radiative diffusion dominates

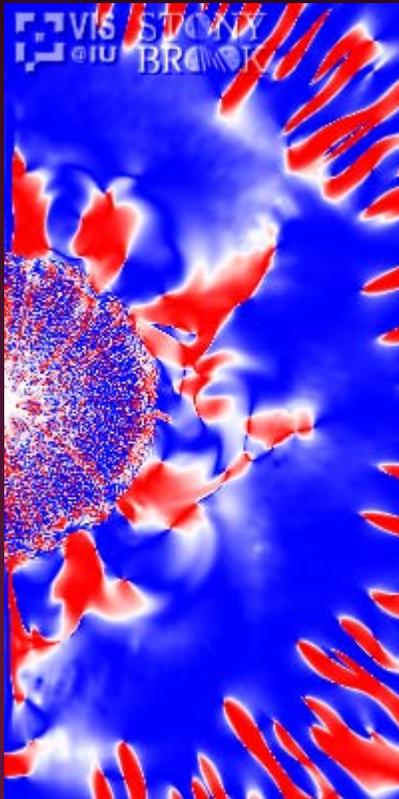


Advective flux

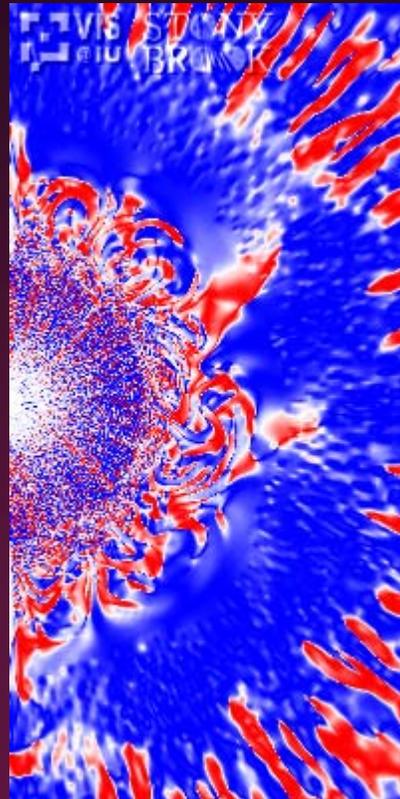
$$\mathbf{F}_\epsilon = \mathbf{v} E_\epsilon$$

$\epsilon_\nu = 1.5 \text{ MeV}$

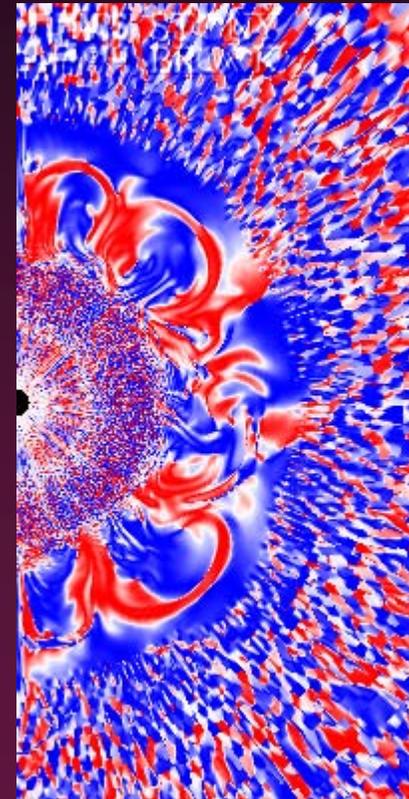
*Radiative & Advective Fluxes behave differently at different neutrino energies. Advective flux does not depend on opacity (which is a function of energy) while radiative flux is strongly opacity dependent.*



$\epsilon_{\nu} = 1.5 \text{ MeV}$



$\epsilon_{\nu} = 18 \text{ MeV}$

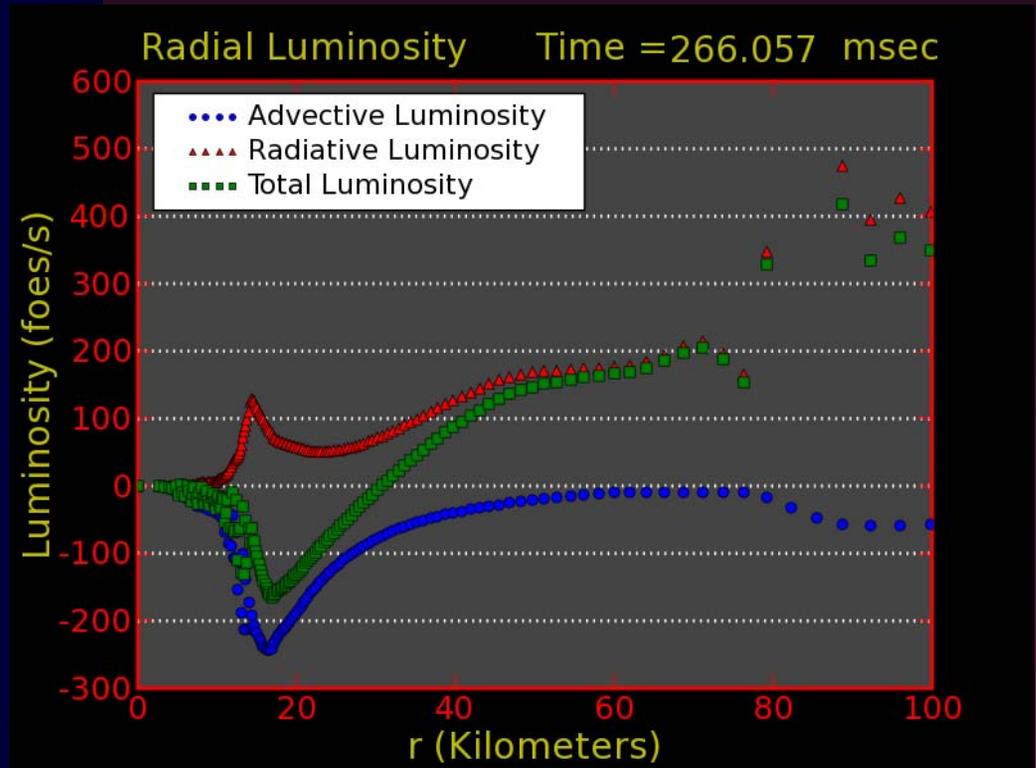


$\epsilon_{\nu} = 380 \text{ MeV}$



## Radial luminosity & convection

- ❑ Convection at the PNS surface gives rise to a fast burst of deleptonization
- ❑ This instability is initiated and driven by an unstable entropy gradient around neutrinosphere
- ❑ Higher  $Y_e$  ( $\sim 0.3$ ) material gets advected upward allowing it to deleptonize around neutrinosphere
- ❑ PNS surface convection enhances neutrino luminosity in a burst but does not maintain this enhancement
- ❑ Dynamic diffusion term  $r\phi(E_\nu v)$  (which has always been neglected) is the leading order term of neutrino transport equation inside of 30 Km!



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# The Future



We now have a computational laboratory to explore effects on nuclear and particle physics in convective epoch of supernovae

- Currently starting to look at effects of nuclear force parameters such as the nuclear symmetry energy, nuclear specific heat & effective masses
- Effects of neutrino electron scattering and other neutrino physics
  - NES models are coming very soon!
- Will be looking at neutrino flavor mixing
- Other progenitor models



Will be extending simulations to later times

- Can the SASI occur in baseline model at later times?



Fully implicit hydro is in development and testing

- Will allow us to solve technical limitations in looking at PNS convection
- Will enable us to calculate neutrino signal



Fully implicit radiation-hydro

- Eliminate operator splitting and allow higher-order time integration



3-D Models with AMR

- Accurately characterize convection
- Will be able to deal with rotation
- Additional goal: estimate gravitational wave signal

# *Thanks to those who have contributed!*

- ❑ TSI Team members:
  - Ed Bacht, Polly Baker (Indiana Univ. @ Indianapolis)
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  - Jim Lattimer (Stony Brook)
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- ❑ SDM ISIC Team members:
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*The Stony Brook TSI web site:*  
<http://nuclear.astro.sunysb.edu>

