Injection in Tokamaks
Adaptive Mesh Simulations of Pellet
Pellet injection: Motivation

- Injection of frozen hydrogen pellets is a viable method of fueling a tokamak
- Presently there is no satisfactory simulation or comprehensive predictive model for ITER
- Ratio of pellet size to device size is ~0.103
- A high Pf plasma is created
- Frozen pellet encounters hot plasma and ablates rapidly
- Non-local electron transport along field lines rapidly heats
- Fast magnetosonic time scale $\tau_{ms}$
- Ions and plasma expands
- So-called "anomalous" transport across flux surfaces $\tau_{a}$
- Pellet cloud expands along field lines $\tau_{p}$
- Equilibration

Physical Processes

- Mass deposition: Large scale MHD driven but not-so-well understood
- Ablation: Considered well-understood
- Pellet-plasma Interactions: Mass deposition: Large scale MHD driven but not-so-well understood
- Presently there is no satisfactory simulation or comprehensive predictive model for ITER
- Ratio of pellet size to device size is ~0.103

Motivation

- Injection of frozen hydrogen pellets is a viable method of fueling a tokamak
<table>
<thead>
<tr>
<th></th>
<th>ITER (Large)</th>
<th>DIII-D (Medium)</th>
<th>CDXu (Small)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points</td>
<td>1.4 x 10^{19}</td>
<td>9 x 10^{17}</td>
<td>7 x 10^{17}</td>
</tr>
<tr>
<td></td>
<td>2.3 x 10^{17}</td>
<td>3.3 x 10^{10}</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>4 x 10^{12}</td>
<td>2 x 10^{7}</td>
<td>0.3</td>
</tr>
<tr>
<td>Spacetime</td>
<td>6.2</td>
<td>1.5 x 10^{11}</td>
<td>CDXu (Small)</td>
</tr>
<tr>
<td>N steps</td>
<td>N</td>
<td>N</td>
<td>Tokamak</td>
</tr>
<tr>
<td>Major Radius</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tokamak</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Resolution Estimates**

- Pellet lifetime: \(\mathcal{O}(10^{-3})\) s → long time integrations
- Large pressure and density gradients in the vicinity of cloud
- Electron heat flux is non-local
- Pellet cloud density: \(\mathcal{O}(10^4)\) times ambient plasma density
- Time scale for reconnection is \(\mathcal{O}(1/\nu)\)
- Thickness of resistive layer scales with \(\mathcal{O}(1/\nu)\)
- Presence of magnetic reconnection
- Spatial scales: Pellet radius \(R_p\) >> Device size \(L \approx \mathcal{O}(10^{-3})\)
- Time scales \(\mathcal{O}(r_c, l_e, l_t, t_e)\)
MHD Primer

- A continuum or fluid description of a plasma
  - A hierarchy of MHD models can be derived from Fokker-Planck equations
  - Single fluid resistive MHD derived
    - Assuming a “single fluid” and quasi-neutrality
    - High collisionality and small Larmour radius

- MHD equations are mathematical models describing the flow of a conducting fluid coupled with electromagnetism

- Governing equations: hydrodynamics coupled with Maxwell’s equations
  - Faraday: $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$ (implies $\nabla \cdot \mathbf{B} = 0$)
  - Ohm: $\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta \mathbf{J}$
  - Ampere: $\mathbf{J} = \nabla \times \mathbf{B}$

- Momentum equations include $\mathbf{J} \times \mathbf{B}$ force
- Energy equation includes $\mathbf{J} \cdot \mathbf{E}$ (Ohmic heating)

- Several ways to combine hydrodynamics with Maxwell
  - Weakly coupled, strongly coupled, vector potential formulations etc.
  - Mathematically, the result is a system of coupled nonlinear partial differential equations which usually must be solved numerically
  - For ideal MHD the equations are hyperbolic PDEs
  - Numerically it is challenging to preserve the solenoidal property of the magnetic field
- Newton-Krylov approach for wide range of temporal scales (not discussed today)
- AMR techniques to mitigate the complexity of the multiphysical scales in the problem

- Electron heating in the neighborhood of the pellet
- Detailed local physics including ablation, ionization and MHD simulations in a tokamak geometry

Current Work
Additional constraint $\nabla \cdot B = 0$.

\[
\begin{align*}
\begin{pmatrix}
\phi_B B_n - \nabla \phi_n \\
0 \\
0 \\
0 \\
0 \\
\phi_B B_n - \phi_B n \\
\nabla \phi_n - \phi_B n \\
0
\end{pmatrix}
\left(\begin{array}{c}
0 \\
\phi_B (n B) - \phi_B (n + e) \\
0 \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n
\end{array}\right)
&= (n) S
\\
\begin{pmatrix}
z B (n B) - z n (n + e) \\
z B \phi_n - z B \\
0 \\
z B B_n - B B \\
z B B_n - B B \\
z B B_n - B B \\
z B B_n - B B
\end{pmatrix}
\left(\begin{array}{c}
0 \\
\phi_B (n B) - \phi_B (n + e) \\
0 \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n
\end{array}\right)
&= (n) H = \mathcal{H}
\\
\begin{pmatrix}
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n
\end{pmatrix}
\left(\begin{array}{c}
0 \\
\phi_B (n B) - \phi_B (n + e) \\
0 \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n \\
\phi_B B_n - \phi_B n
\end{array}\right)
&= (n) F = F
\end{align*}
\]

Energy: Electron heat flux
Density: Ablation

Diffusive terms

Hyperbolic terms

Single fluid resistive MHD equations in conservation form

Mathematical Model
Advection velocity is \( q \). 

Solve by using an upwind method. 

Solve for opacities as a "steady-state" solution to an advection-reaction equation. 

Assumes Maxwellian electrons and neglects pitch angle scattering.

Electron Heat Flux Model
Equations in transformed coordinates

\[ \mathbf{S} = \mathbf{q} \]
\[ H^u R - \mathbf{F}^u \zeta = (H^z + \mathbf{F}^z) \mathbf{f} = \mathbf{f} \]
\[ H^3 \zeta + \mathbf{F}^3 \zeta = (H^z u + \mathbf{F}^z u) \mathbf{f} = \mathbf{f} \]
\[ S = \frac{\phi e}{\zeta e} \frac{H}{H} + \frac{ue}{He} \frac{R}{R} + \frac{u \zeta}{h \zeta} \frac{R}{R} + \frac{u \zeta}{h \zeta} \frac{R}{R} \]

Curvilinear coordinates for shaped plasma

- Coordinate \( \phi \) is retained as before
- \( u \equiv u \) and \( \zeta \equiv \zeta \)
- \( R \equiv R \) and \( Z \equiv Z \)
- Flux surfaces are determined from a separate equilibrium calculation

Adopt a flux-tube coordinate system
Boundary Conditions: Perfectly conducting BCs, periodic in $\phi$.

Solver.

Initial state is an MHD equilibrium obtained from a Grad-Shafranov expression $B = 1/R(\phi)$.

Initial Conditions: $\Delta \times \phi \neq Jc(\phi)$.

Imposing the solenoidal condition on $B$ is important.

Differences.

Diffusive fluxes computed using standard second order central:

$$F = \nabla \cdot (\chi \nabla u)$$


- Seven-wave Riemann solver. $F = F(\nu, \nu') = \frac{1}{2}(F(u) + F(u'))$.
- Explicit second order or third order TVD Runge-Kutta time stepping.

Finite volume approach.

Numerical Method.
Time step ensures conservation
Flux-resetting step at end of
and areas of faces upon refinement
Mesh generation: necessary to ensure volume preservation
Adaptivity in both space and time

(Chombo is an AMR developer’s toolkit)
(http://www.seesar.lbl.gov/ANAG/chombo)

Difference calculations
block-structured adaptive mesh refinement (AMR) finite

Chombo is a collection of C++ libraries for implementing
Adaptive Mesh Refinement with Chombo
Pellet Injection: AMR
Pellet Injection: Zoom into Pellet Region
Pellet Injection: Zoom in
Results - HFS vs. LFS
Results - HFS vs. LFS
Results - HFS vs. LFS
Most likely explanation: Nonlinear manifestation of an MHD interchange instability.

Arrows indicate average pellet location

HFS pellet injection shows better core fueling than LFS vs. LFS - Average Density Profiles
Pellet Injection: LFS/HFS Launch

Density
between levels combine communication and irregular computation.

Interlevel operations: Interpolating boundary data, averaging / interpolation
disjoint union of rectangles to some other union of rectangles.
Communication primitives: Exchange of ghost cell data, copying from a
access only to local data.
local computation: Iterate over patches owned by processor. Processor has
data built on top of these metadata. Distributed grid
processors have access to processor assignment metadata. All
Domain decomposition that assigns rectangular patches to processors. All

AMR Programming Model
Defining Scalability, Performance

Performance: Efficiency is very close to the fraction of operator peak computation. For the examples described here, implementation directly: what fraction of time is spent on regular runs and assuming perfect weak scaling, rather than computed AMR calculation. The former is generally estimated from smaller uniform grid at the finest resolution to the time to solution for the adaptation factor: ratio of the time to perform the calculation on a performance: We assess performance in terms of a fraction of operator peak. Typically 10%-20% of the nominal peak. For stencil an operator on a uniform grid on a single processor. For stencil operators, performance: maximum performance for evaluating performance.
Scalability of AMR for explicit methods is relatively easy.

Adaptivity factor: 16.

Fraction of operator peak: 85% (450 Mflops / 8192 processors (173-181 seconds).

96% efficient scaled speedup over range of 128- box "from Chombo distribution.

Results obtained with hyperbolic code "out of the

regidding, etc.

Timing only the update step - no initialization,

Operator peak performance on XT4 is 530 Mflops / processor.

coarse time step.

points, with 1B grid point updates performed per
points. 16x16x16 (6) patches, five unknowns / cell, 62M grid
proportional refinement in space, fixed-sized
refinement, factor of 4 each. Refinement in time
levels of

inert grid covering a spherical shell. Two levels of

singled image is a spherical shock tube in 3D, with

interpolated only once per update. Easiest case.

cell. Explicit method, so ghost cell values copied /

Unsplit PPM solver - 6K fops / grid point to update AMR Gas Dynamics Benchmark.
AMR Poisson Benchmark

- Development of scalable Poisson solvers is one of the most challenging goals for AMR.
- Fraction of operator peak: 45% (375 Mflops / 8.192 processors (8.4-9.5 seconds)).
- 87% efficient scaled speedup over range of 256-14,336 processors.
- Improvement in per-processor performance and scalability.
- Optimization (2 months) leading to 10X.
- Results obtained after significant effort in code.
- Timing only the solver - no initialization.
- Mflops / processor.
- Operator peak performance on XT4 is 840.
- Image.
- Unknown per cell, total of 15M grid points per cell.
- Allowed to vary between 8 and 32.
- One refinement factor of 4 each. Patch size is 2 levels of 8.
- Single image is two rings. Two rings of elliptic solvers on AMR grids.
- 100 calls to co-iteration. Typical of broad range of over 1000 calls to co-iteration / exchange.
- Over 100 calls to communication (exchange) of AMR-MG V-cycle = 1700 floppy / grid point.
- Multilevel discretization of Laplacian with AMR.
Future Work

Adaptivity factor estimated for these simulations ~ 100-200

Simulations show better core fueling for HFS injection

Developed a finite volume upwind adaptive curvilinear

Conclusions
Department of Energy under Contract No. DE-AC03-
Supported by the Office of Science of the U.S.
Research Scientific Computing Center, which is
This research used resources of the National Energy
No. DE-AC02-76-CH03073
SciDAC Centers. RS supported by US DOE contract
Funded through the TOPS, CEMM and ADEC
(TOPS)
D. Reynolds (UCSD, TOPS), C. Woodward (LLNL,
S. C. Jardin (CEMM, PPL)
(ADEC, LBNL)
P. Collella and Applied Numerical Algorithms Group
Acknowledgement