QCD Thermodynamics at High Temperature

Peter Petreczky

NY Center for Computational Science

Large Scale Computing and Storage Requirements for Nuclear Physics (NP)
Bethesda MD, April 29-30, 2014
Defining questions of nuclear physics research in US:
“What are the phases of strongly interacting matter and what roles do they play in the cosmos?”
“What does QCD predict for the properties of strongly interaction matter?”

Challenges for LQCD:
1) Equation of state
2) Phase diagram and the transition temperature
3) Fluctuations of conserved charges
4) In-medium hadron spectral functions
5) Transport coefficients
Deconfinement at high temperature and density

Hadron Gas

Transition

Quark Gluon Plasma (QGP)

Color screening

magnetic screening, non-perturbative

Deconfinement at high temperature and density

\[ T_c = (154 \pm 9) \text{ MeV} \]

Chiral symmetry is broken in the low temperature (hadronic) “phase”

but is restored at high \( T \)

QCD analog of ferromagnet-paramagnet transition

\[ \alpha_s(T) \ll 1 \]

Chiral critical end-point
Physics of heavy ion collisions and LQCD

- High temperature QCD
- Weak coupling?
- EM and heavy flavor probes
- Chiral transition, $T_c$ fluctuations of conserved charges
- EoS, viscosity
- Test of Hadron Resonance Gas (HRG) using LQCD
- Quarkonium spectral functions, heavy quark diffusion, thermal dileptons
Finite Temperature QCD and its Lattice Formulation

\[ \langle O \rangle = \text{Tr} O e^{-\beta H - \mu N} \]

\[ \langle O \rangle = \int \mathcal{D} A_\mu \mathcal{D} \psi \mathcal{D} \overline{\psi} O e^{-\int_0^\beta d\tau d^3x \mathcal{L}_{QCD}} \]

\[ A_\mu(0, x) = A_\mu(\beta, x) \quad \psi(0, x) = -\psi(\beta, x) \]

Lattice

\[ \langle O \rangle = \int \prod_x dU_\mu(x) O(\text{det} D_q[U, m, \mu]) e^{-\sum_x S_G[U(x)]}, U_\mu(x) = e^{iga A_\mu(x)} \]

Generation of gauge configuration (Markov chain) \( \mu = 0 \) \( \rightarrow \) Hybrid Monte-Carlo

Most of cycles : \( D_q^{-1} A \)

\( \mu \neq 0 : \text{det} D_q(U, m, \mu) \) complex \( \rightarrow \) sign problem \( \rightarrow \) Taylor expansion for not too large \( \mu \)

Most of cycles : \( D_q^{-1} A \)

- Highly Improved Staggered Quark (HISQ) relatively inexpensive numerical but does not preserve all the symmetries of QCD (except for zero \( a \))
- Domain Wall Fermion (DWF) formulation: preserves all the symmetries but costs \( \sim 100x \) of staggered formulation

Need to take the limit of zero \( a \) : cost \( \sim 1/a^7 \), \( T = 1/(aN_\tau) \)
- rapid change in the number of degrees of freedom at $T=160\text{--}200\text{MeV}$: deconfinement
- deviation from ideal gas limit is about 10\% at high $T$ consistent with the weakly coupled quark gluon gas
- free energy of static quark anti-quark pair shows Debye screening $\Rightarrow$ quarkonium suppression @RHIC
HotQCD: a collaborative effort

**Software (USQCD):**

- **MILC code** (MIMD C code + platform dependent optimization at t lower level) also for GPU (CUDA)
- **Columbia Physics System, CPS** (C++, optimized for BG/Q, DWF only)
- Optimized CUDA GPU code from Bielefeld U.

**Resource in 2013-2014 (in core h):**

1. CPU clusters of USQCD: 40M
2. GPU clusters of USQCD: 2.5 M
3. INCITE allocation of USQCD: 120M (Mira)
   - 44M (Titan, projected)
4. BNL, BG/Q: 18M, BG/L: 50M,
5. GPU cluster at LLNL and Bielefeld U.: 3.1M
6. BG/Q in Europe: 15M
7. NERSC allocation (PI Bazavov): 10M
8. BG/Q (LLNL): 100 M (DWF thermo only)
QCD thermodynamics at non-zero chemical potential

Taylor expansion:

\[
p(T, \mu_B, \mu_Q, \mu_S) = \frac{1}{T^4} \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi_{ijk} B_{ij} Q_{jk} S_{kl} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_S}{T}\right)^k
\]

LQCD : Taylor expansion coefficients → fluctuations of conserved charges: \(X = B, S, Q\)

Beam energy scan @ RHIC → parameters of the distribution

\[
\begin{align*}
\chi_1^X &= \frac{1}{V T^3} \langle N_X \rangle \\
\chi_2^X &= \frac{1}{V T^3} \langle (\delta N_X)^2 \rangle \\
\chi_3^X &= \frac{1}{V T^3} \langle (\delta N_X)^3 \rangle \\
\chi_4^X &= \frac{1}{V T^3} \left[ \langle (\delta N_X)^4 \rangle - 3 \langle (\delta N_X)^2 \rangle^2 \right]
\end{align*}
\]

\[
M_X = \langle N_X \rangle \quad \text{(mean)}
\]

\[
\sigma_X = \langle (\delta N_X)^2 \rangle \quad \text{(variance)}
\]

\[
S_X = \langle (\delta N_X)^3 \rangle / \sigma_X^3 \quad \text{(Skewness)}
\]

\[
K_X = \langle (\delta N_X)^4 \rangle / \sigma_X^4 - 3 \quad \text{(Kurtosis)}
\]

\[
N_X = X - \bar{X}, \quad \delta N_X = N_X - \langle N_X \rangle
\]

can calculated very effectively on single GPUs

Volume independent combinations:

\[
\begin{align*}
M_X / \sigma_X &= \chi_1^X / \chi_2^X \\
S_X \cdot \sigma_X &= \chi_3^X / \chi_2^X \\
K_X \cdot \sigma_X^2 &= \chi_4^X / \chi_2^X
\end{align*}
\]
Experimental knowledge of strange and charm hadron spectrum is rather incomplete. Future experiments @ Jlab and FAIR (Germany) will address this problem.

HRG that includes hadron states predicted by quark model (also LQCD) agrees better with lattice results than HRG with PDG states only.
QCD thermodynamics at non-zero chemical potential


For consistent description of
The freeze-out of strange hadrons
Need to include the contribution of
“missing states”
Bazavov et al, arXiv:1404.6511
Spectral functions at $T>0$ and physical observables

\[
G(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}
\]

Heavy meson spectral functions:

\[J_H = \bar{\psi} \Gamma_H \psi\]

quarkonia properties at $T>0$

heavy quark diffusion in QGP: $D$

Heavy flavor probes at RHIC

Light vector meson spectral functions:

\[J_\mu = \bar{\psi} \gamma_\mu \psi\]

Thermal photons and dileptons provide information about the temperature of the

thermal dilepton production rate

\[
\frac{dW}{d\omega d^3p} = \frac{5\alpha_{em}^2}{27\pi^2} \frac{1}{e^{\omega/T} - 1} \frac{\sigma_{\mu\mu}(\omega, p, T)}{\omega^2 - p^2}
\]

thermal photon production rate:

\[
p \frac{dW}{d^3p} = \frac{5\alpha_{em}}{9\pi} \frac{1}{e^{p/T} - 1} \sigma_{\mu\mu}(\omega = p, p, T)
\]

electric conductivity $\zeta$:
Quarkonium spectral spectral functions

Charmonium spectral functions on isotropic lattice in quenched approximation with Wilson quarks:

$N_t=24-96$, $\alpha_s^{-1}=18.97\text{GeV}$

No clear evidence for charmonium bound state peaks above $T_c$ in spectral functions!
Lattice calculations of transport coefficients

Electric conductivity:

Ding et al, PRD 83 (11) 034504

peak at $\omega \approx 0 = \text{transport peak}$

$$\Gamma \sim 1/\tau_{\text{relax}}, \quad \sigma_{\text{el}} \sim \chi Q / \Gamma$$

Heavy quark diffusion constant:

Ding et al, arXiv:1204:4954
Banarjee et al, arXiv:1109.5738
Kaczmarek et al, arXiv:1109:3941

Quenched QCD calculations up to $128^3 \times 32$ lattice

Perfect liquid: $\eta/s = 1/(4 \pi) \Leftrightarrow \text{no transport peak!}$
Summary

- Nature of QCD transition
- Transport coefficients
- Di-lepton, photon rate
- QCD critical point
- Di-lepton, quarkonia melting
- Freeze-out conditions
- EOS with dynamical charm quark
- EOS w/o dynamical charm quark
- QCD at non-zero density on coarse lattices

Timeline:
- ~2012
- ~2017
- ~2022

Sustained Petaflop-Years
Back-up slide

MILC code weak scaling

\begin{align*}
(L=4, \times) \ (6, \odot) \ (8, \odot) \ (10, \times) \ (12, +) \ (14, \odot) \\
CG \ GF \ FF \ LL(fat) \\
\end{align*}